Microeconomic Modeling of Incentives for Managed Overlays

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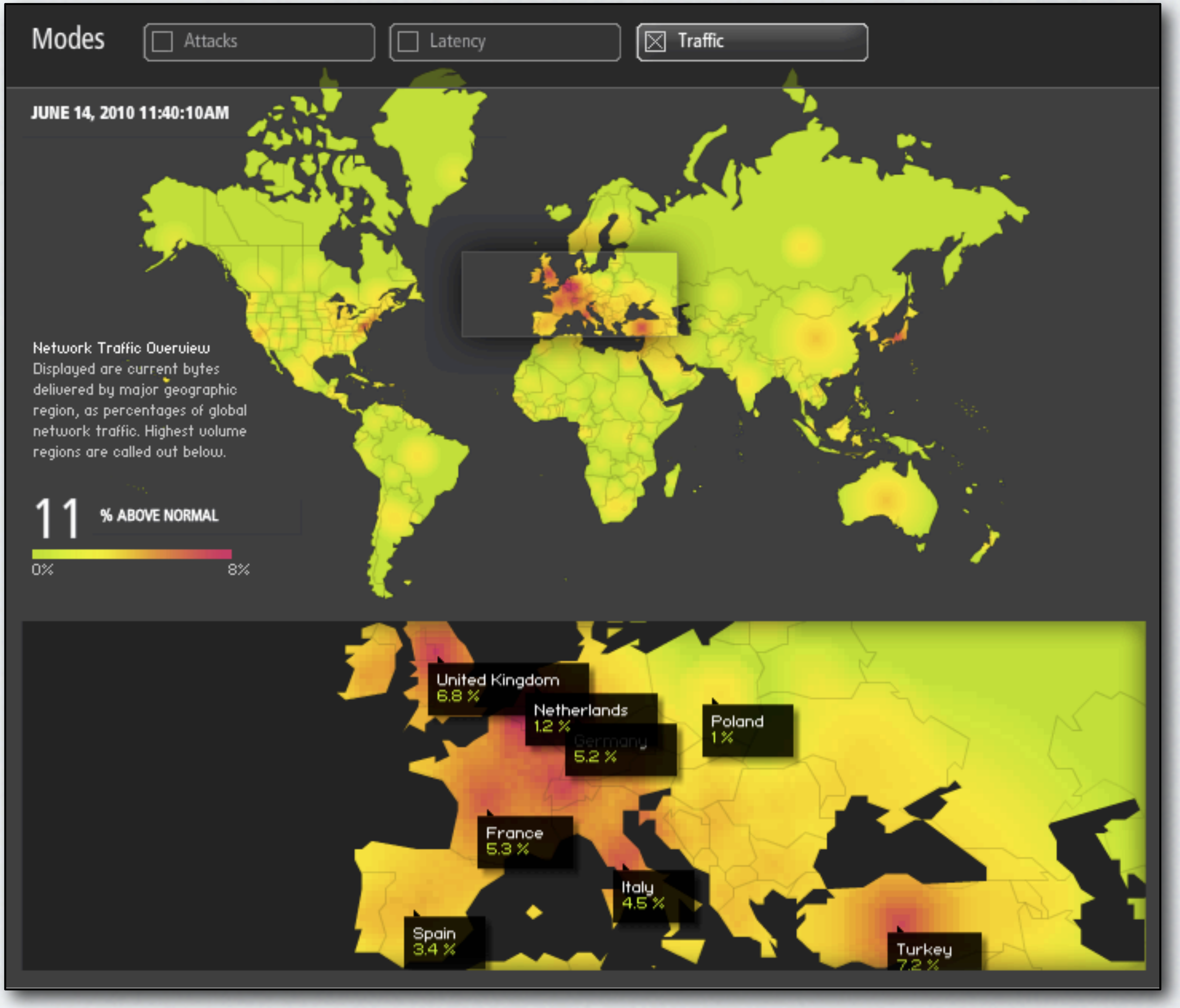
An ISP Market for Managed Overlays

- Application-Layer Overlays are increasingly important in the provisioning of high QoS service
    - Akamai
    - Limelight
    - KonTiki
    - Skype
    - BitTorrent
    - etc...



Network Traffic Overview
Displayed are current bytes delivered by major geographic region, as percentages of global network traffic. Highest volume regions are called out below.

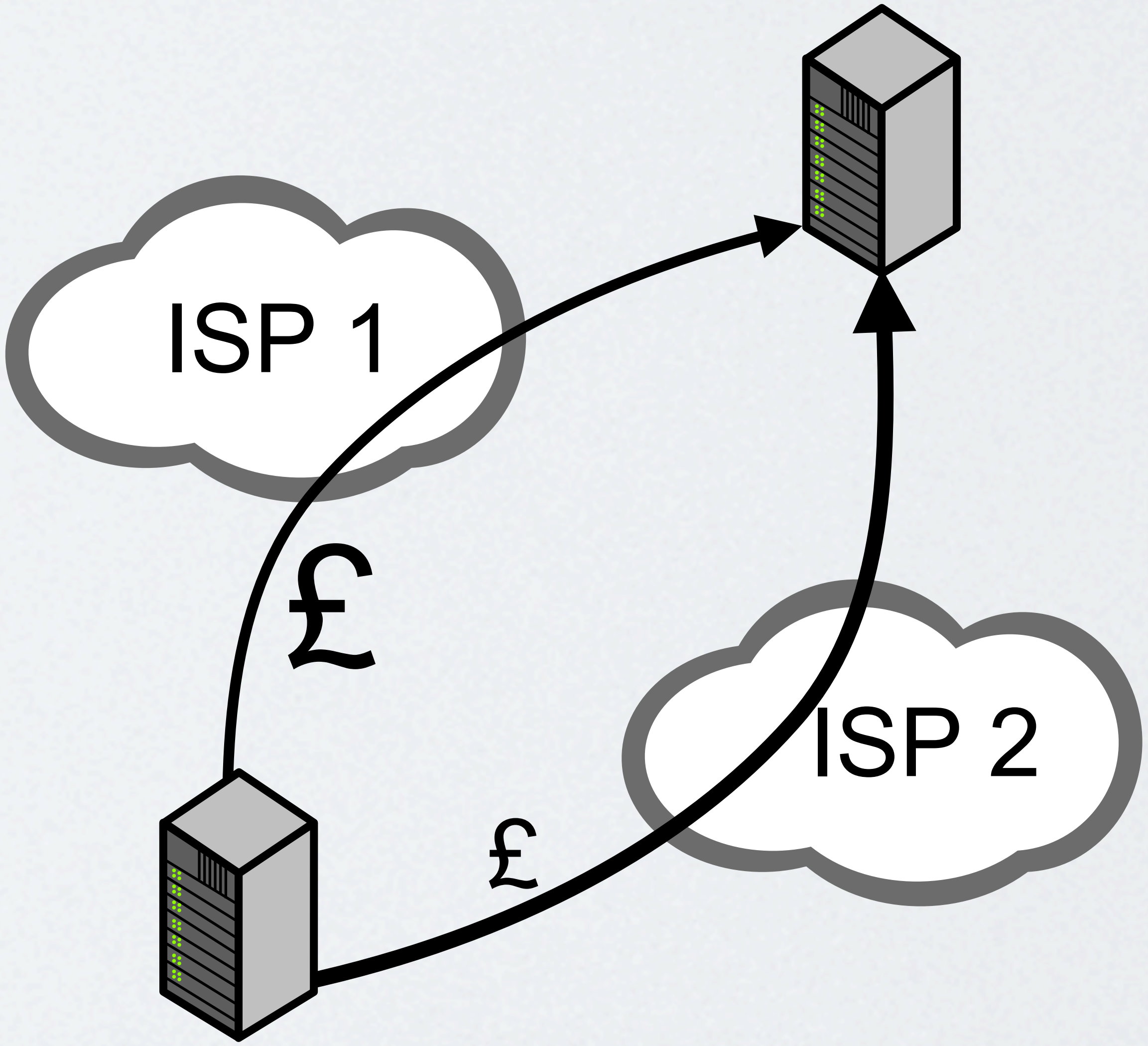
http://www.akamai.com/html/technology/dataviz1.html

An ISP Market for Managed Overlays

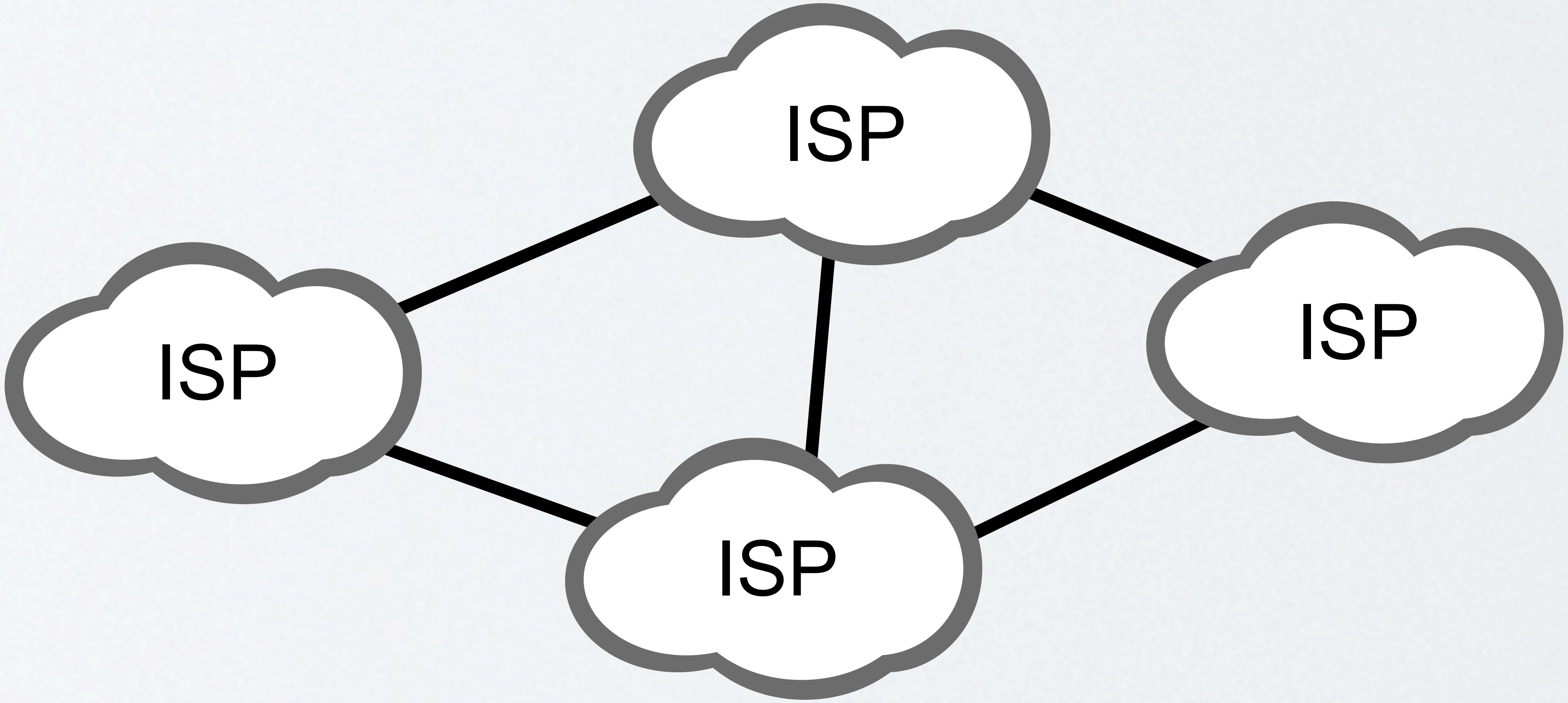
- ***IP Multihoming*** is particularly attractive for end nodes of these kinds of services

- There is a trend towards usage-based billing

- Can we model a market where overlays dynamically allocate loads among ISPs on the basis of edge-to-edge instantaneous pricing?

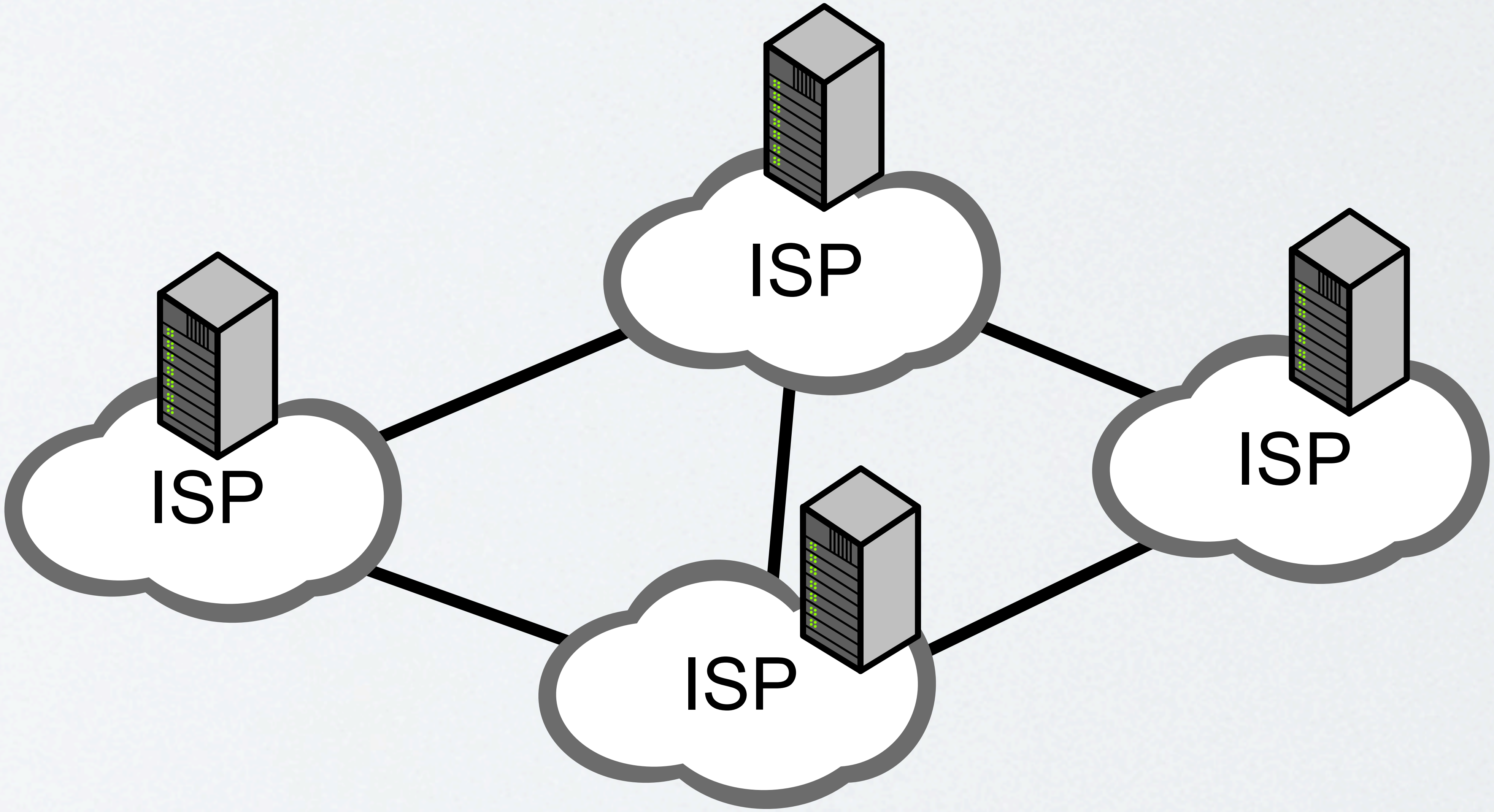


Overlay Providers



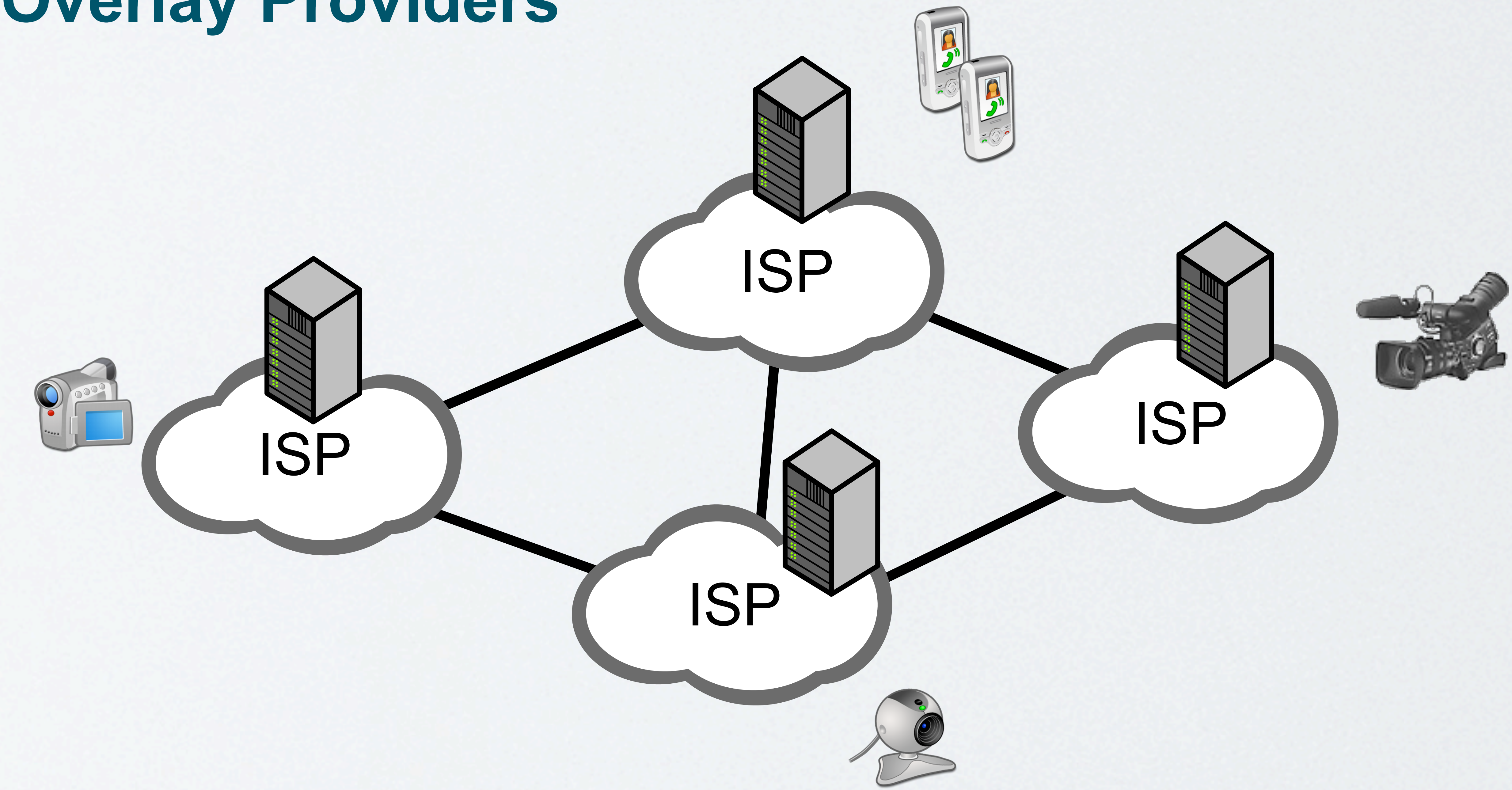
We focus on ISPs that provide access links, as opposed to transit operators

Overlay Providers



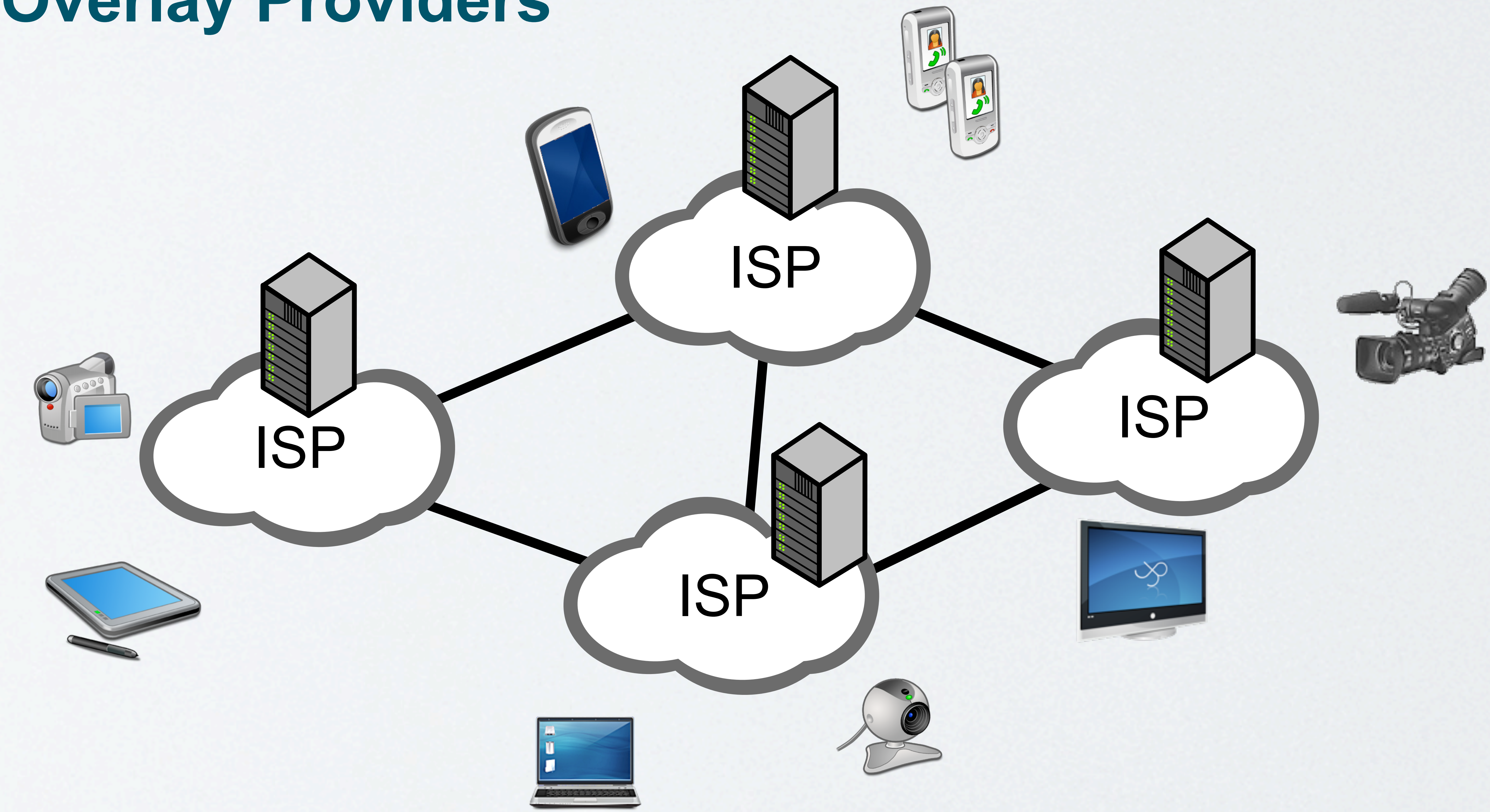
Edge Service Providers (ESPs) deploy managed nodes at the network edge

Overlay Providers



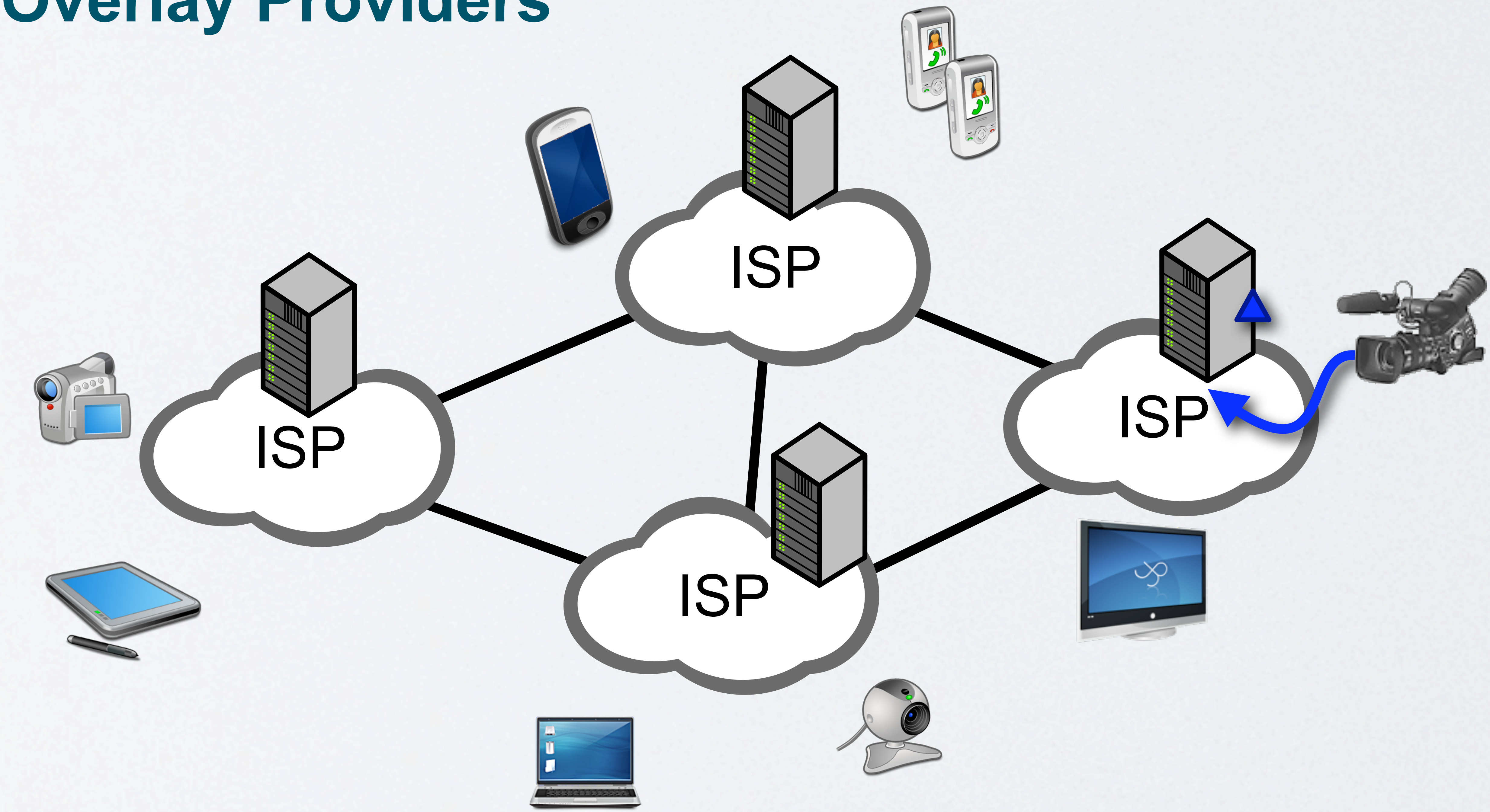
Content can be **generated** anywhere at the network access

Overlay Providers



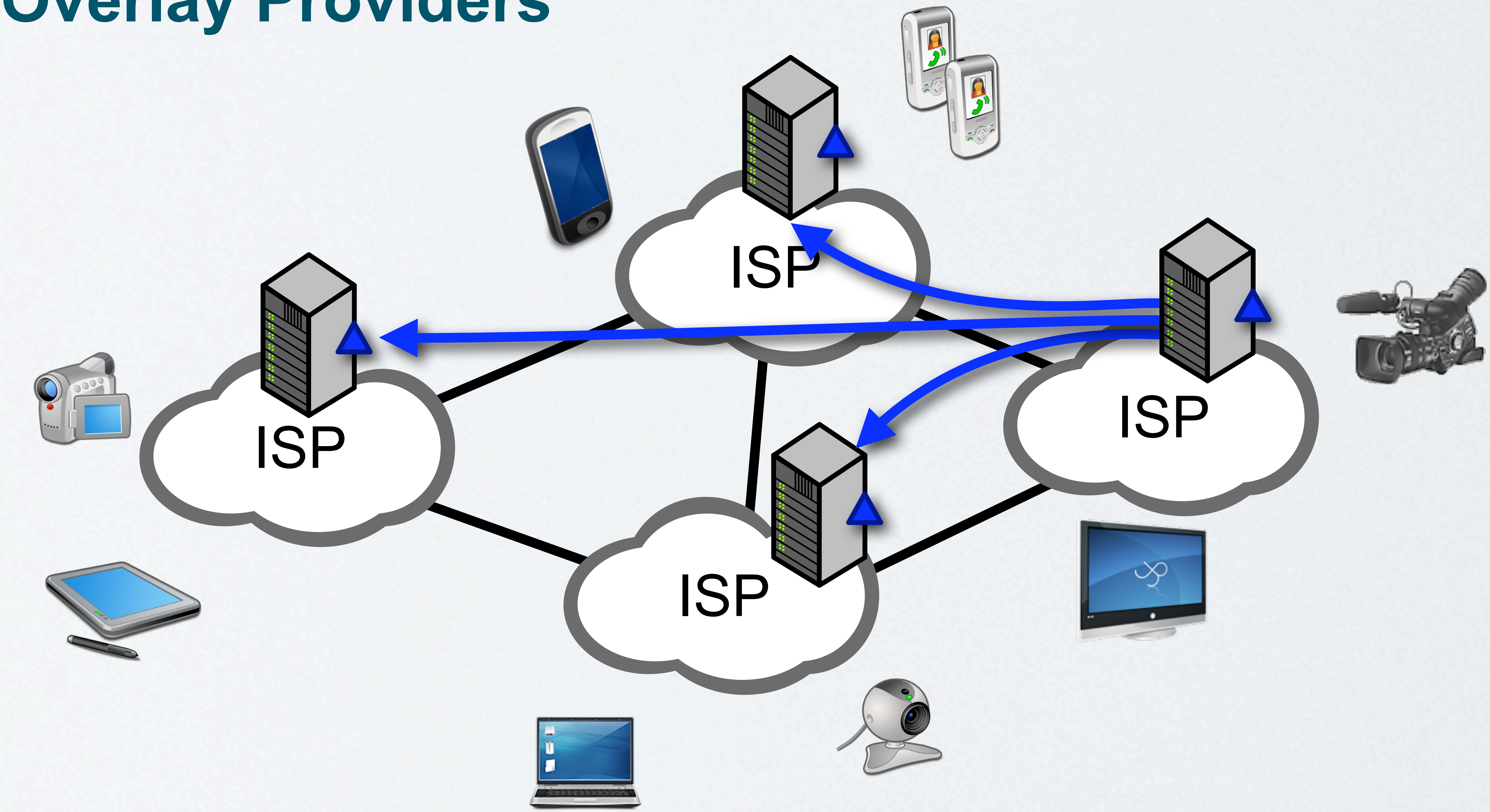
Content can be **consumed** anywhere at the network access

Overlay Providers



Content is **distributed** by the managed overlay

Overlay Providers



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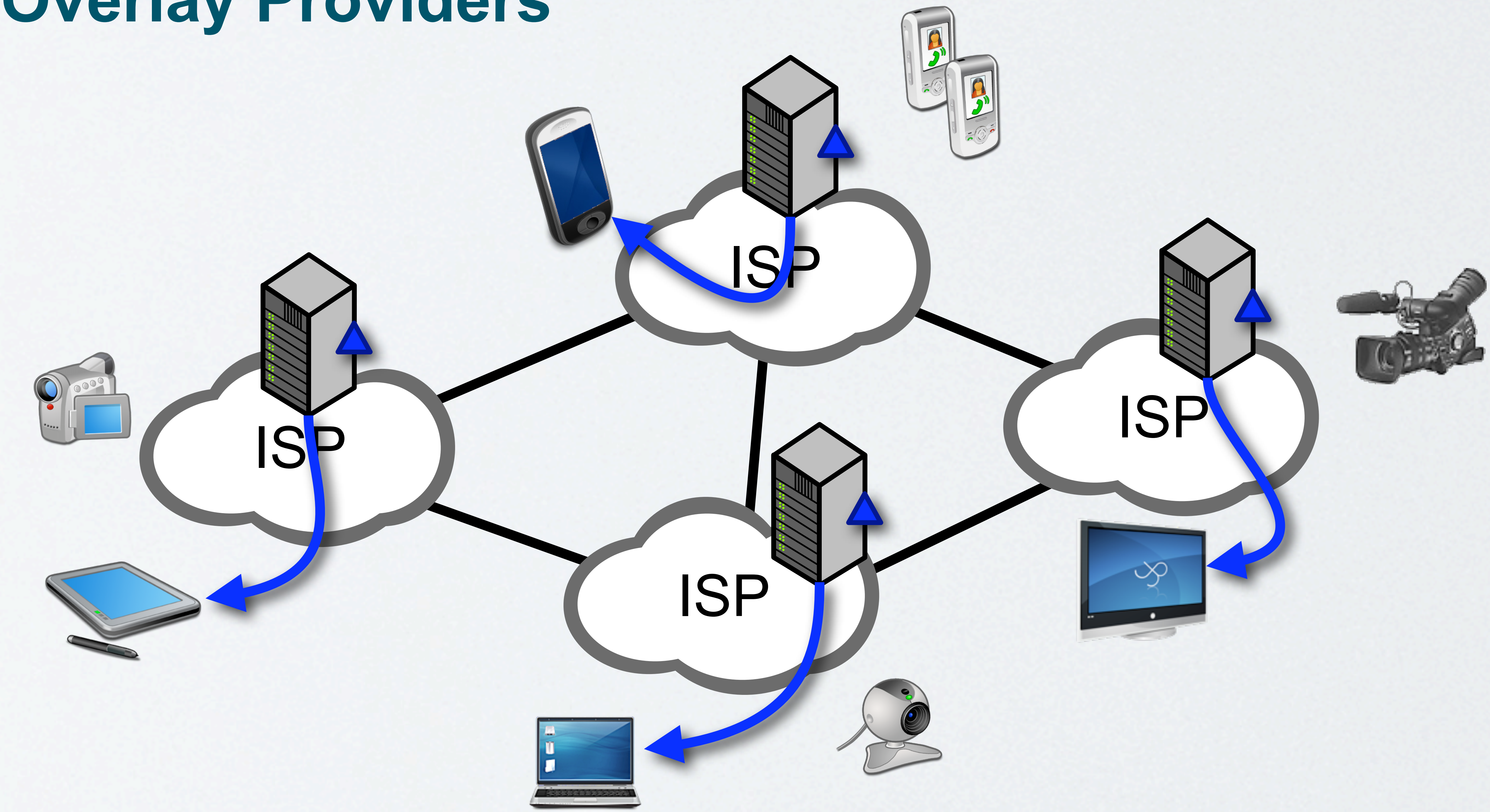

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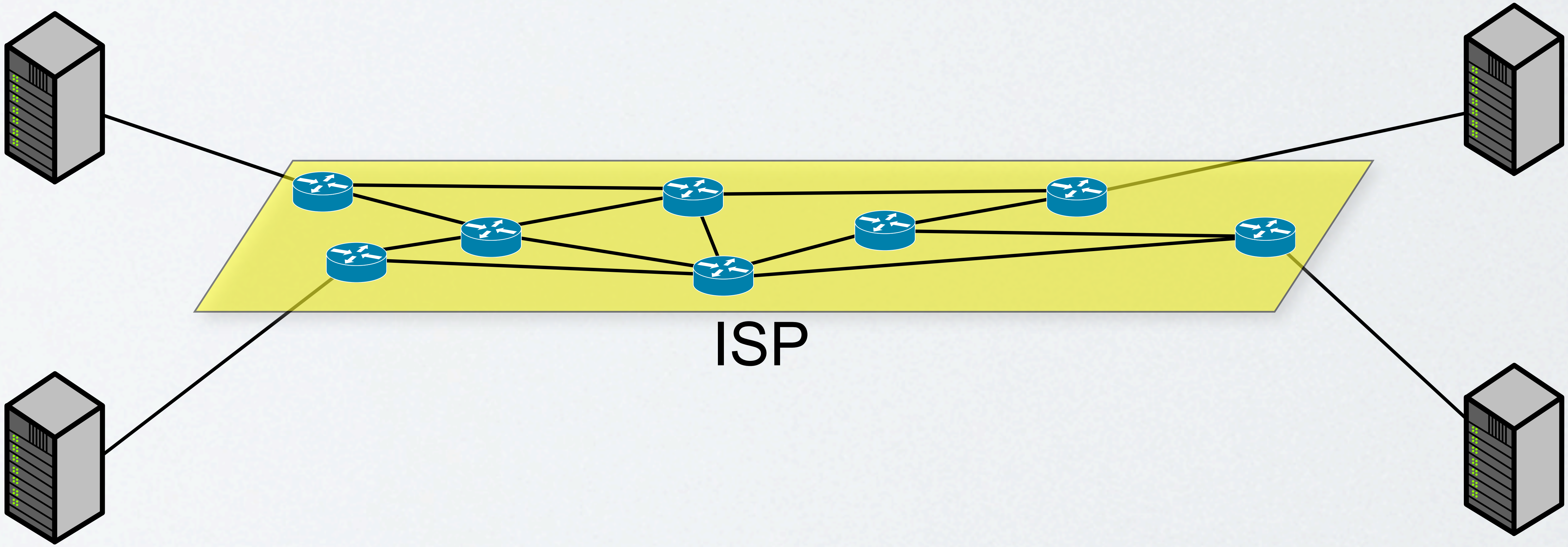

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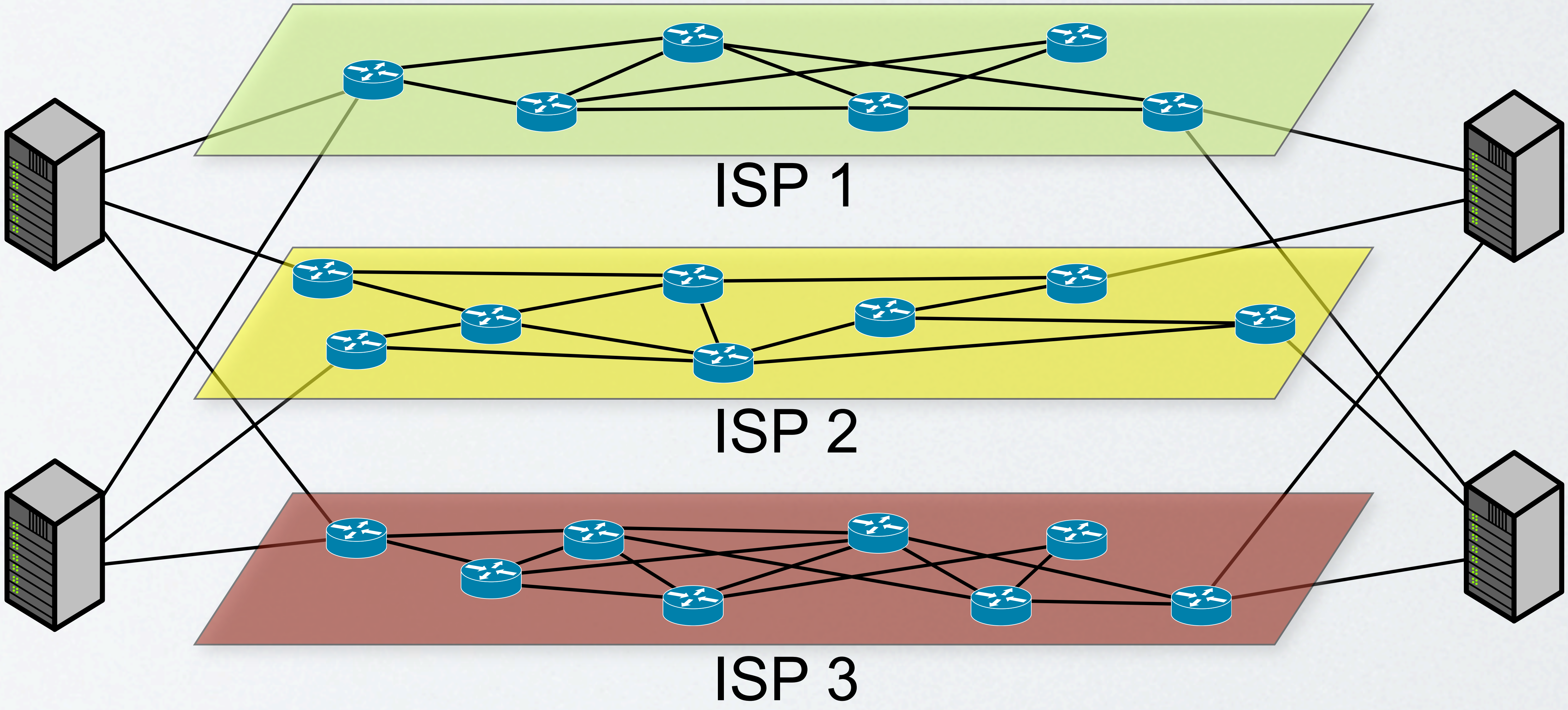

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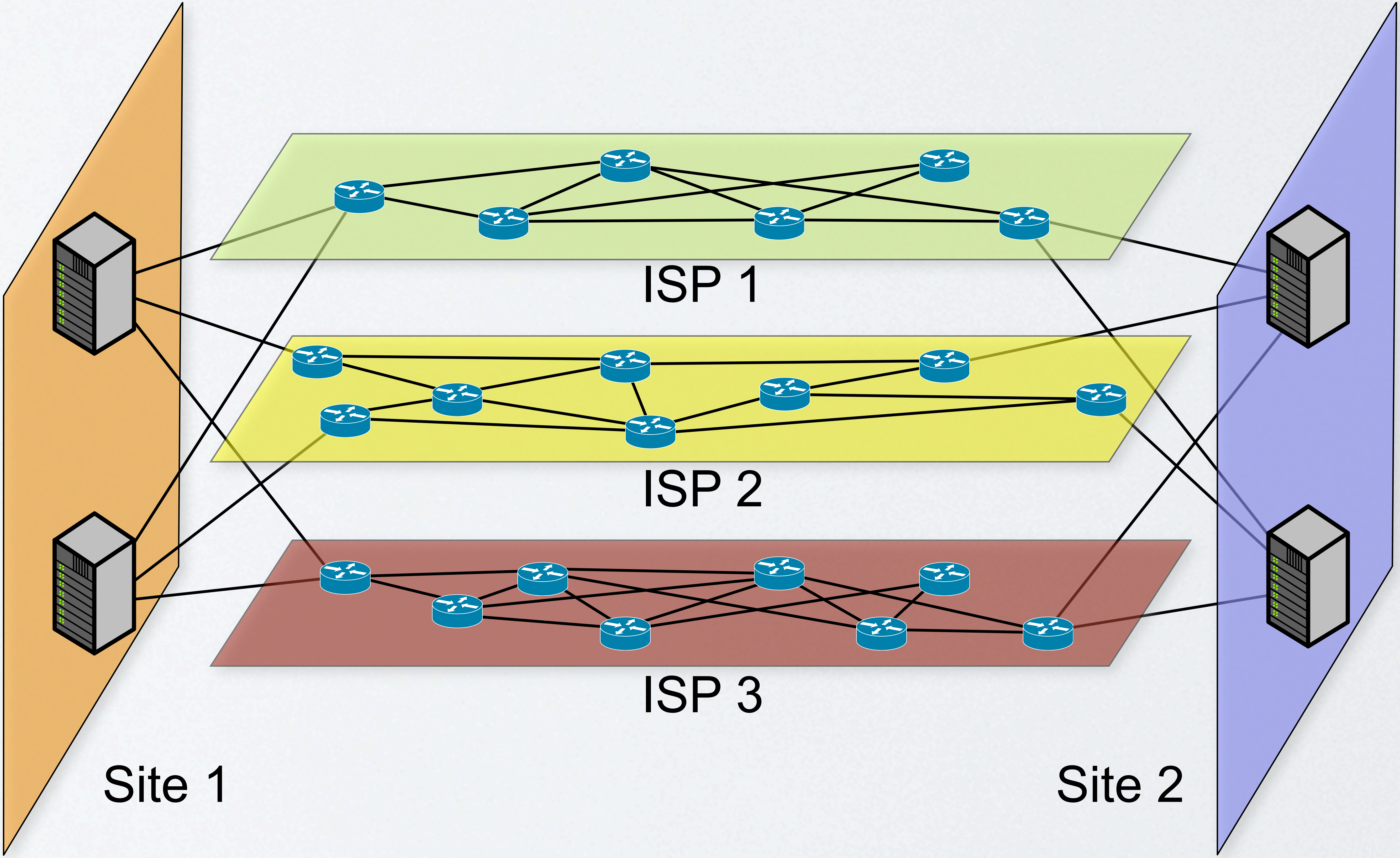

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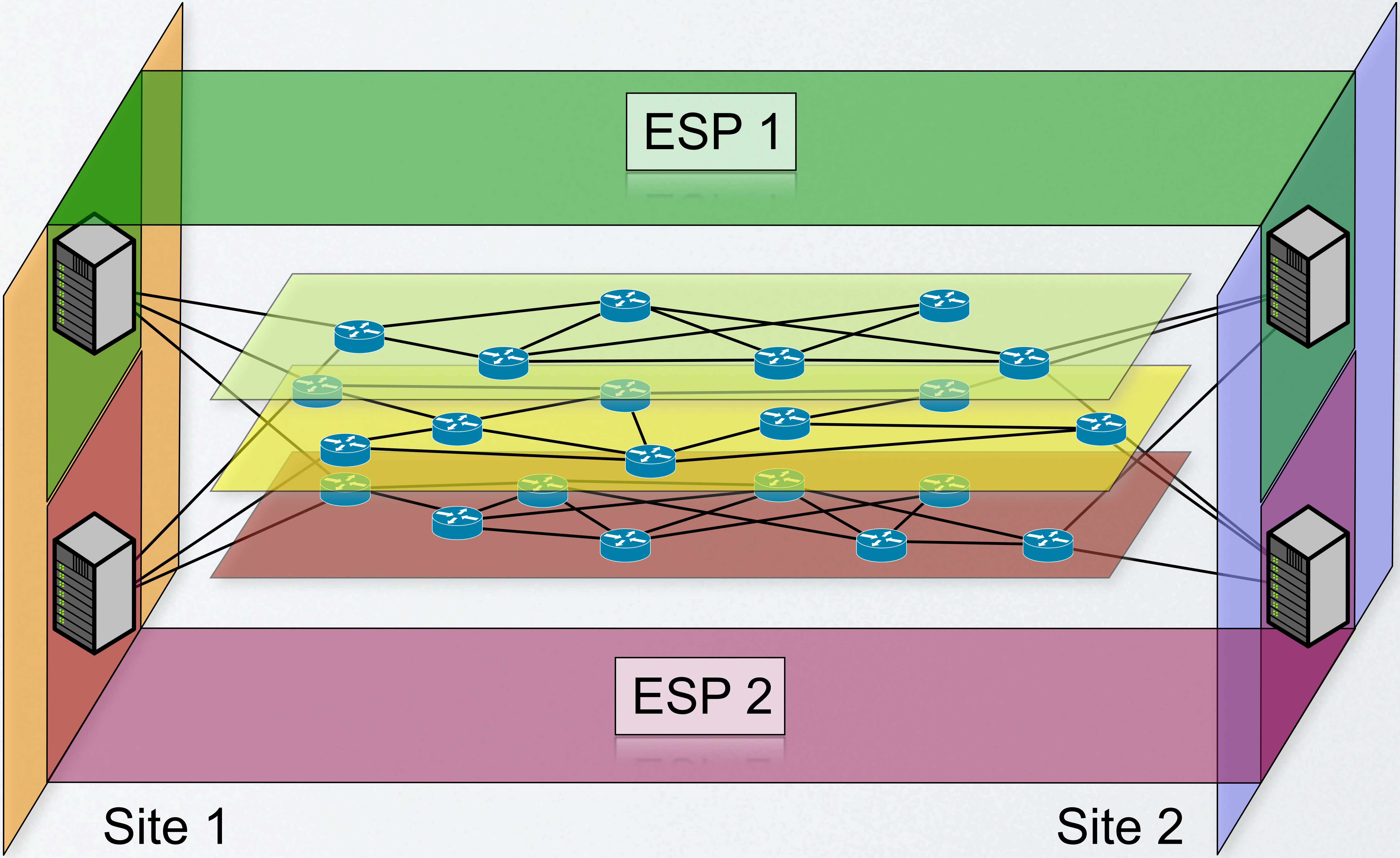
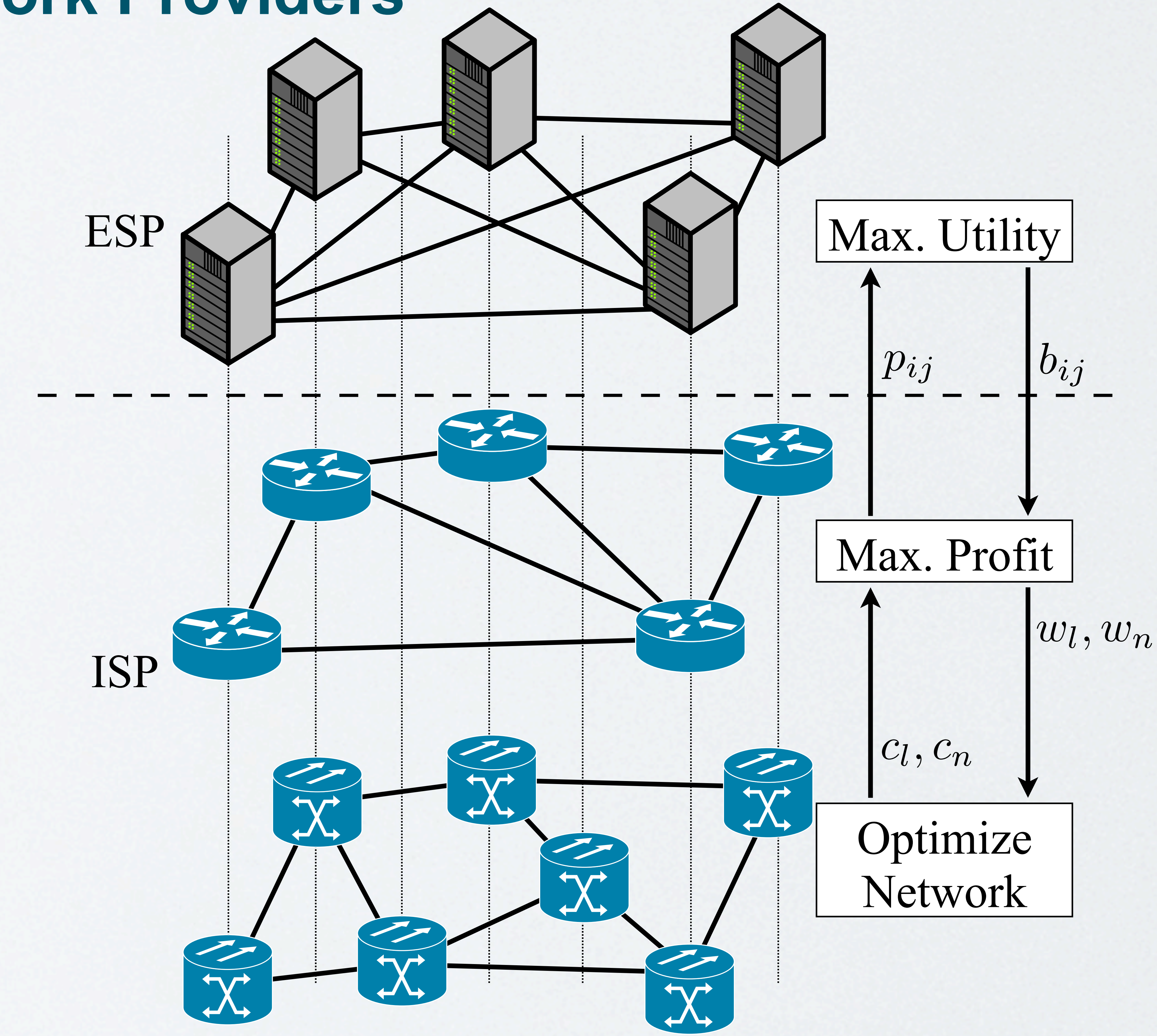
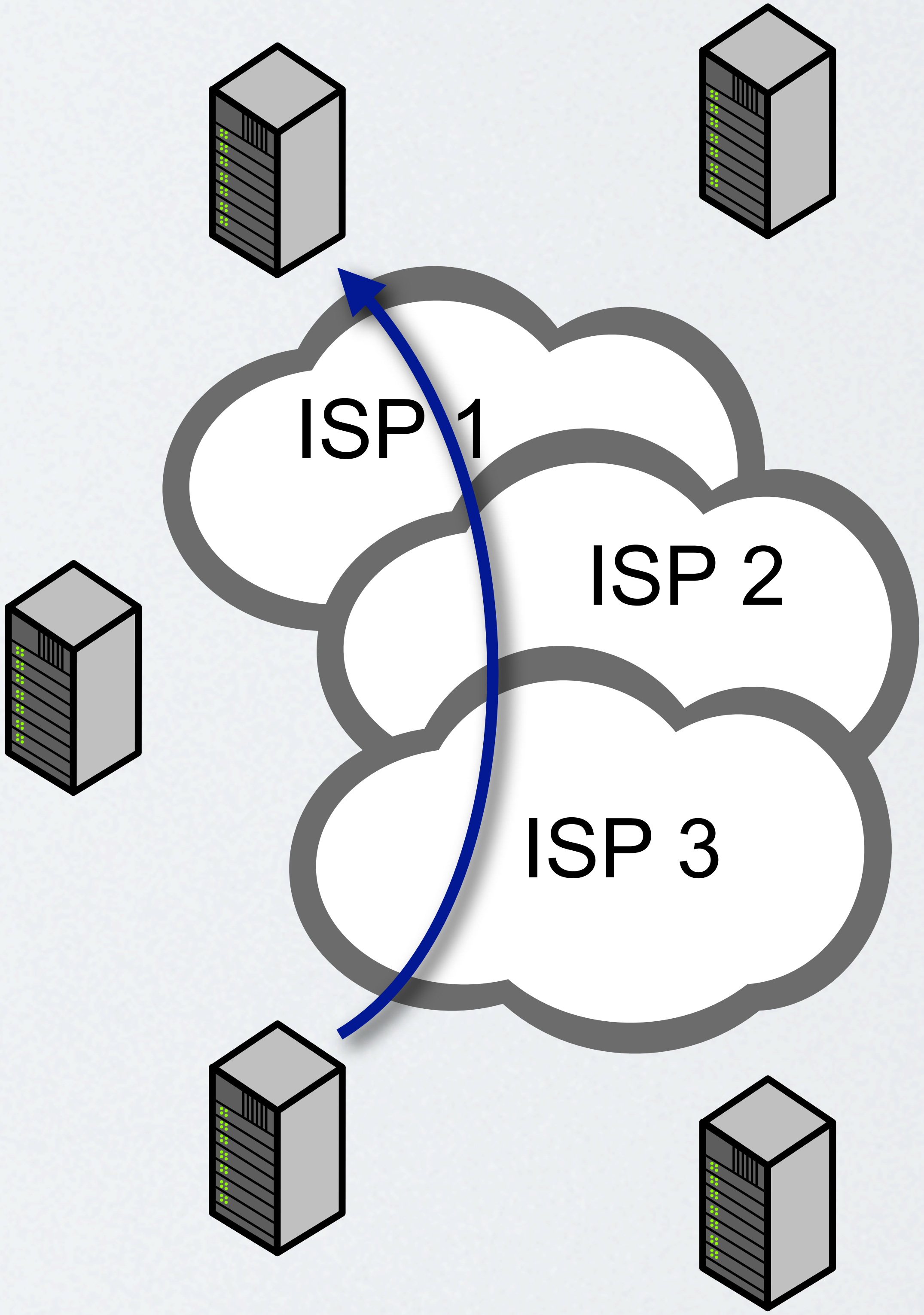


Network Providers



Modeling Overlay Preferences

- Increasing utility with increasing traffic exchange between any two sites
  - Simplest case: replicating all traffic at every site
  - Cost limitations prevent this



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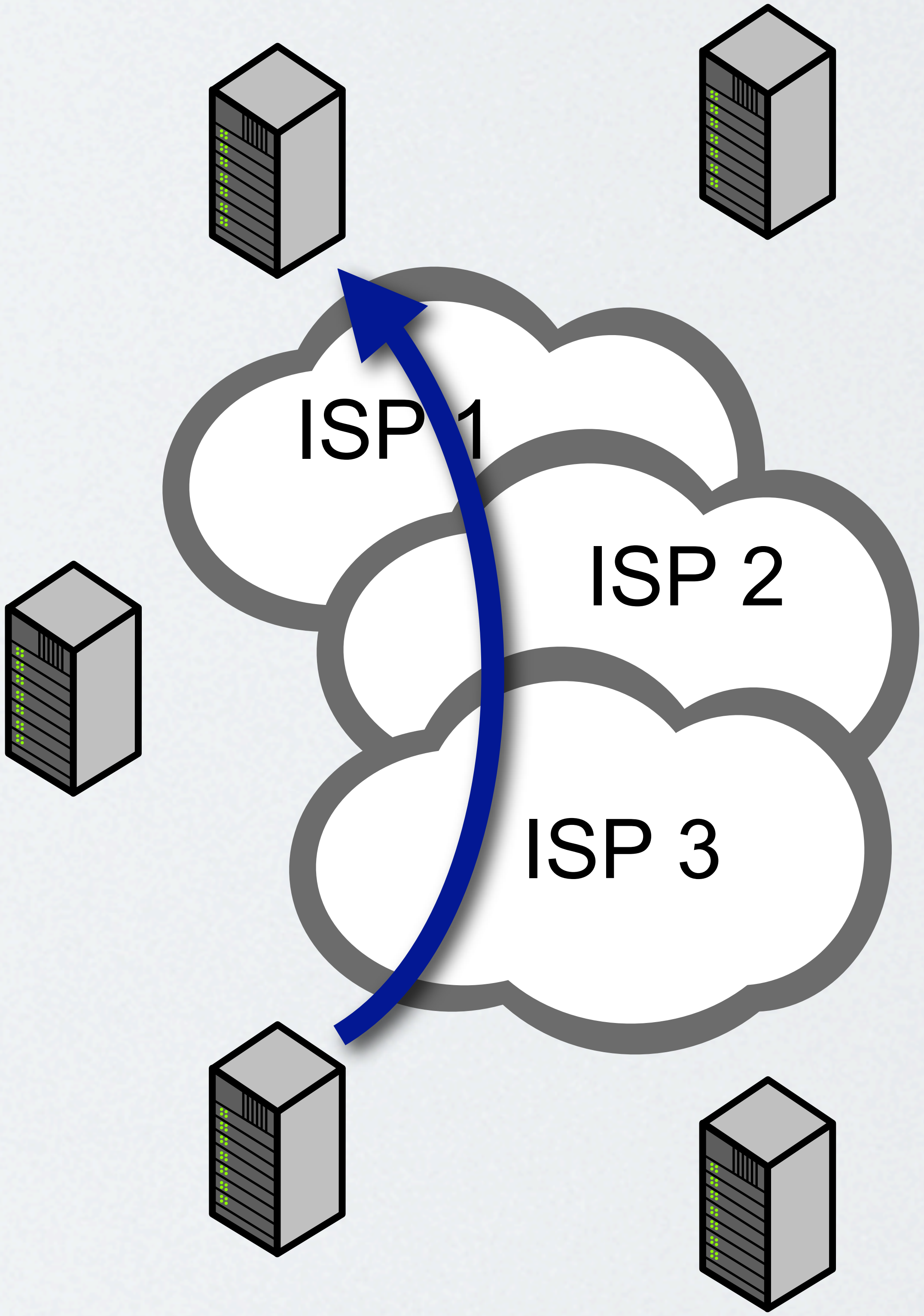
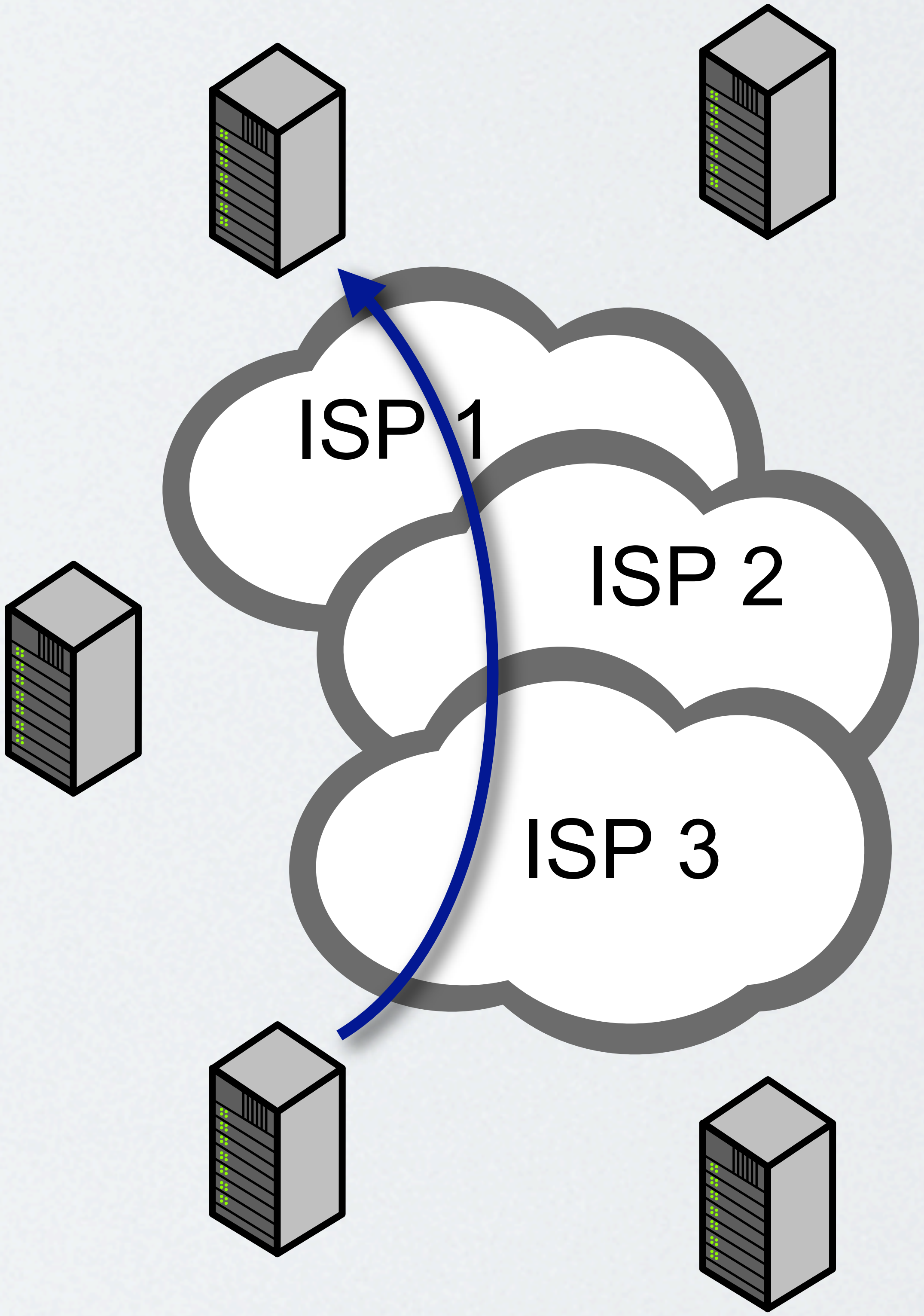


Modeling Overlay Preferences

- Increasing utility with increasing quality on exchanges between any two sites
  - Overlay links will be annotated with some notion of quality q_{ski}
  - Transferring a given amount of traffic between two sites yields greater utility if the quality of the overlay link between them increases



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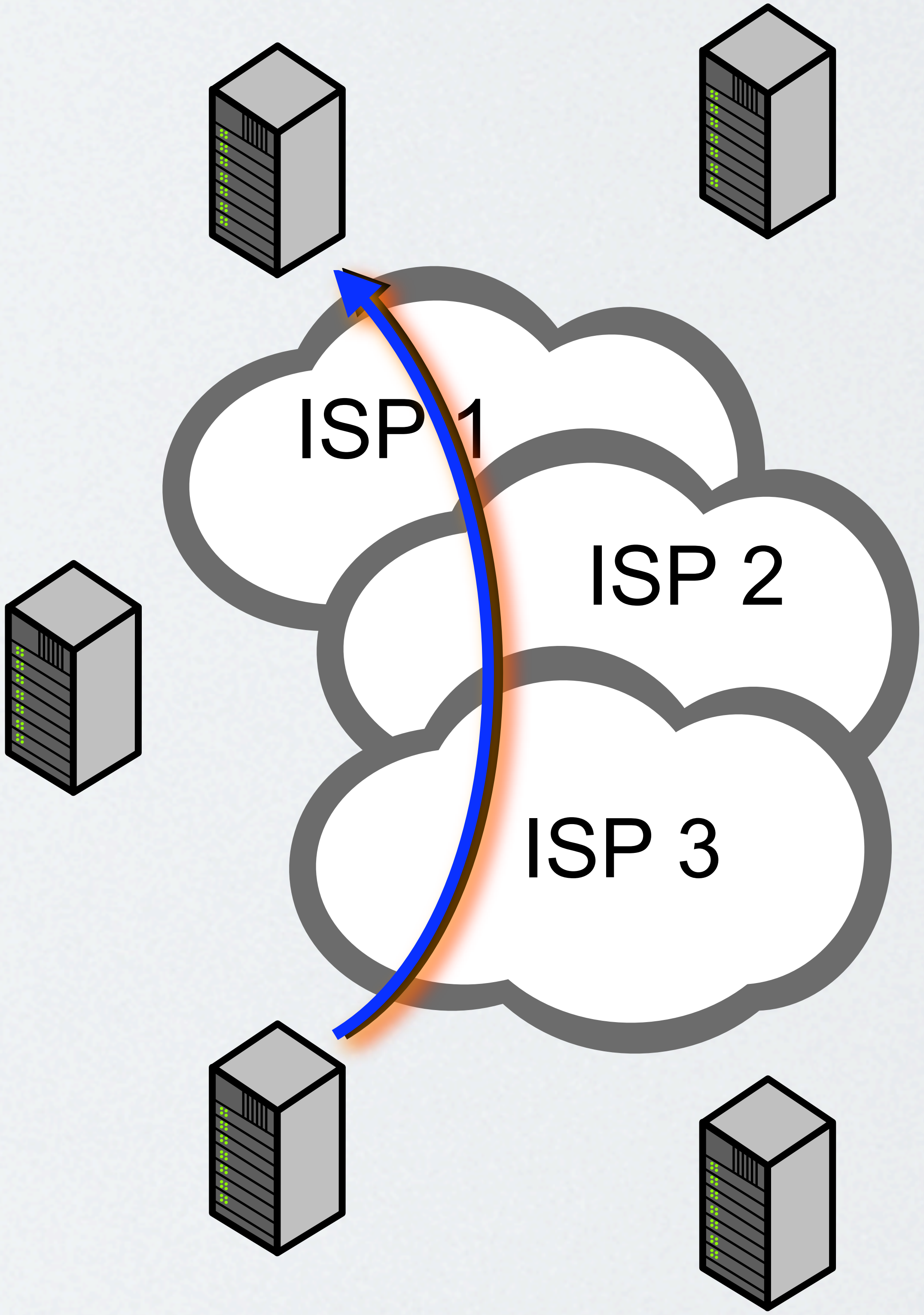
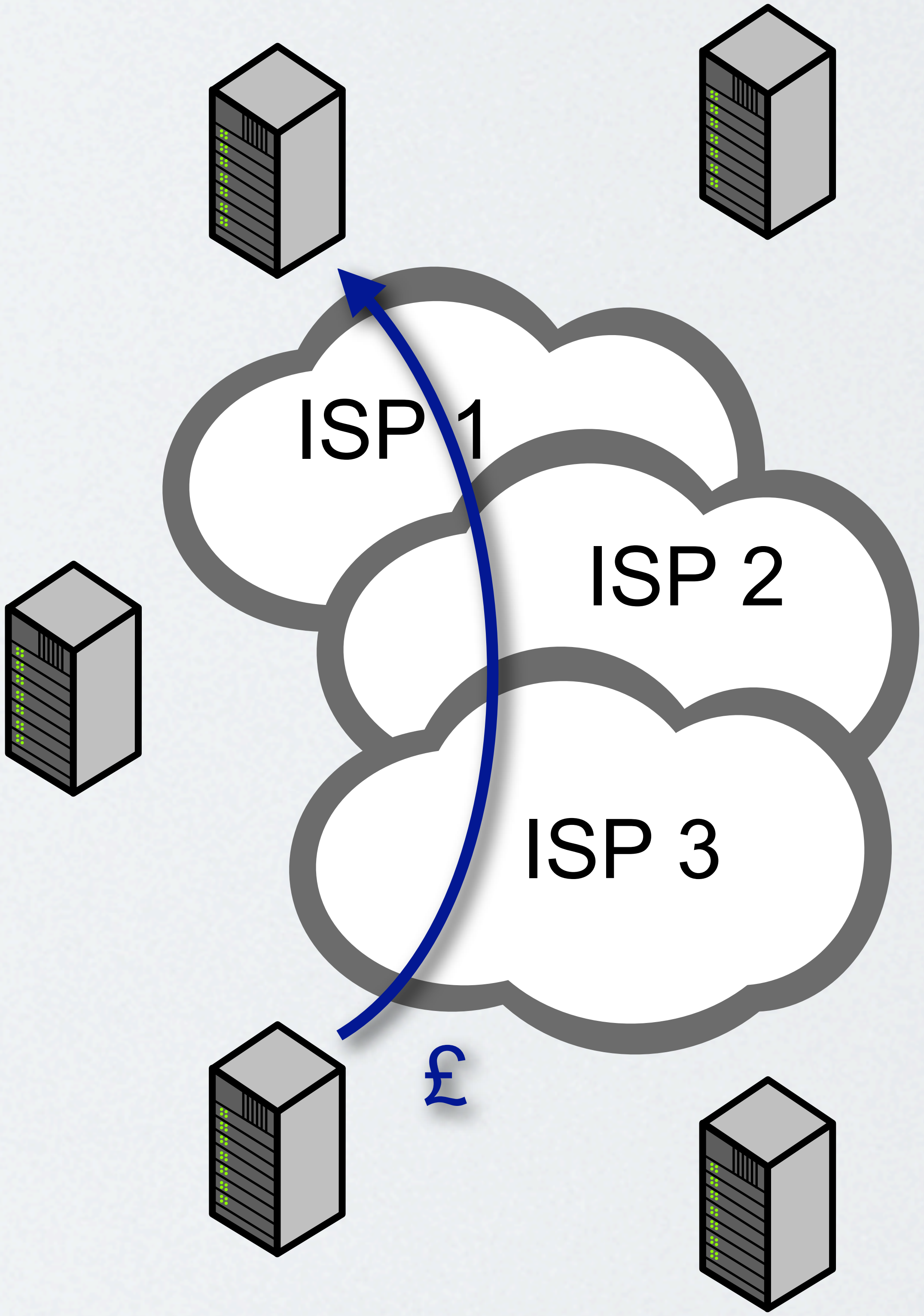


Modeling Overlay Preferences

- ESPs pay an increasing cost with increasing traffic volume exchanged between any two overlay sites
  - We assumine a simple pay-per-volume model



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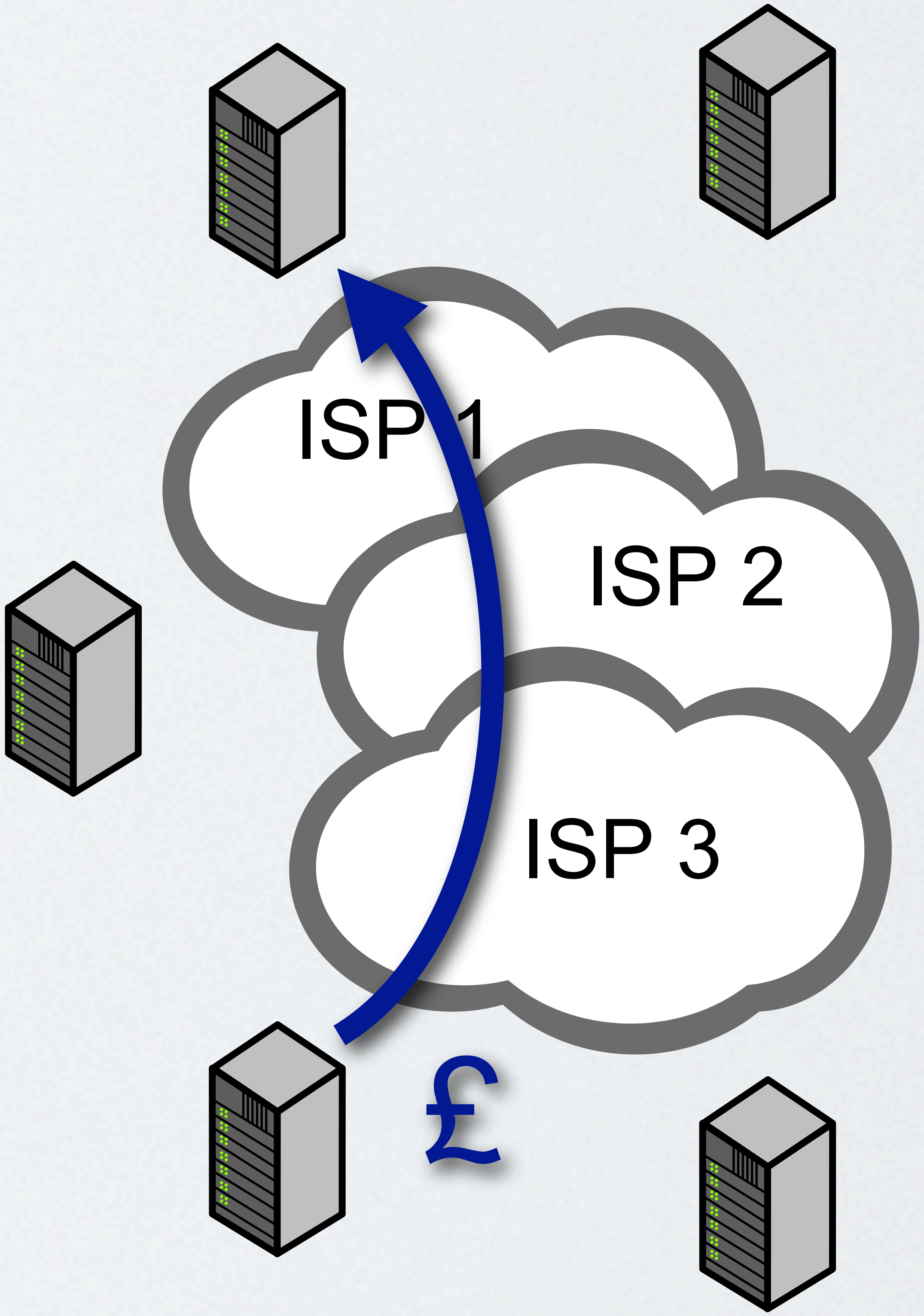
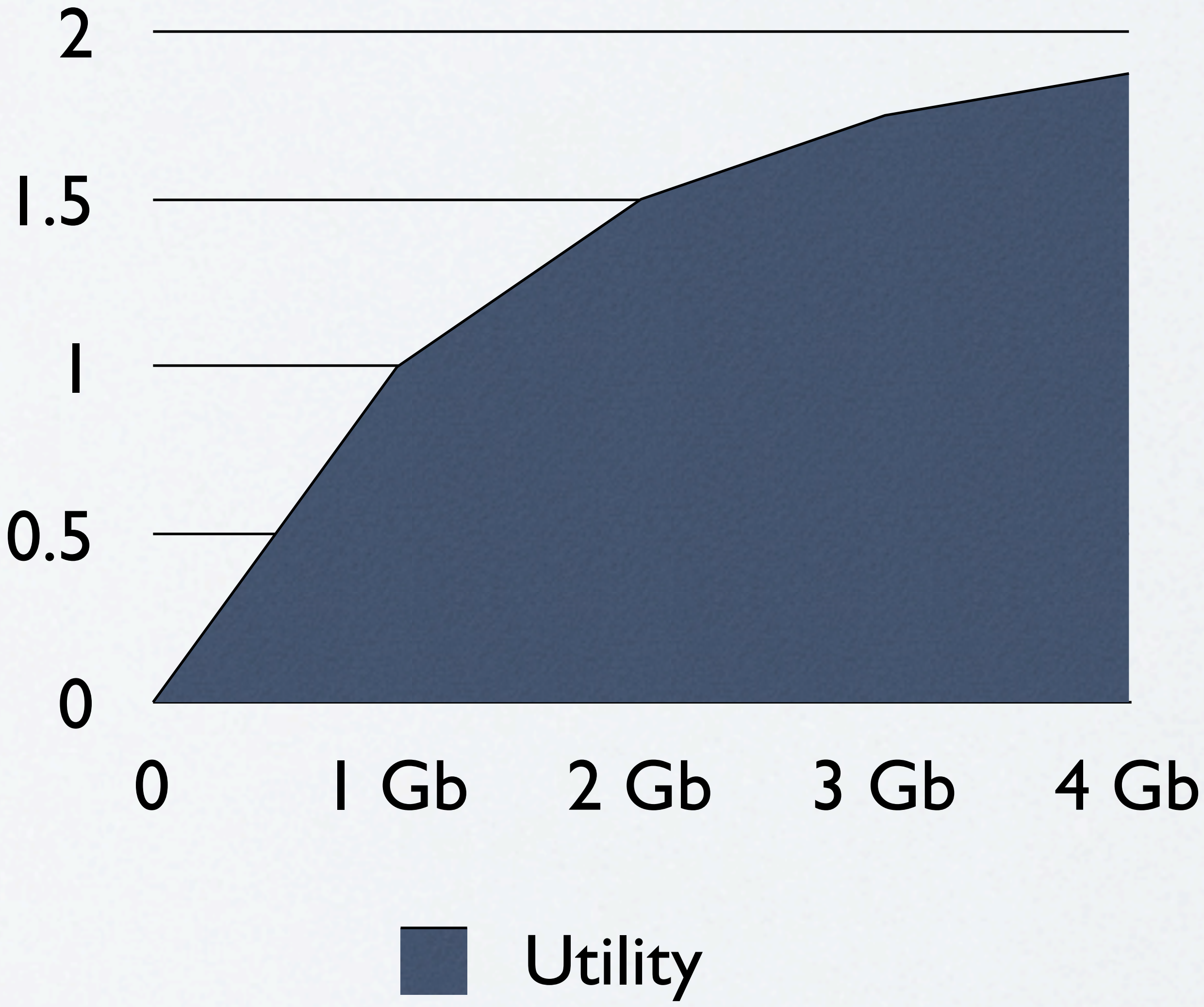
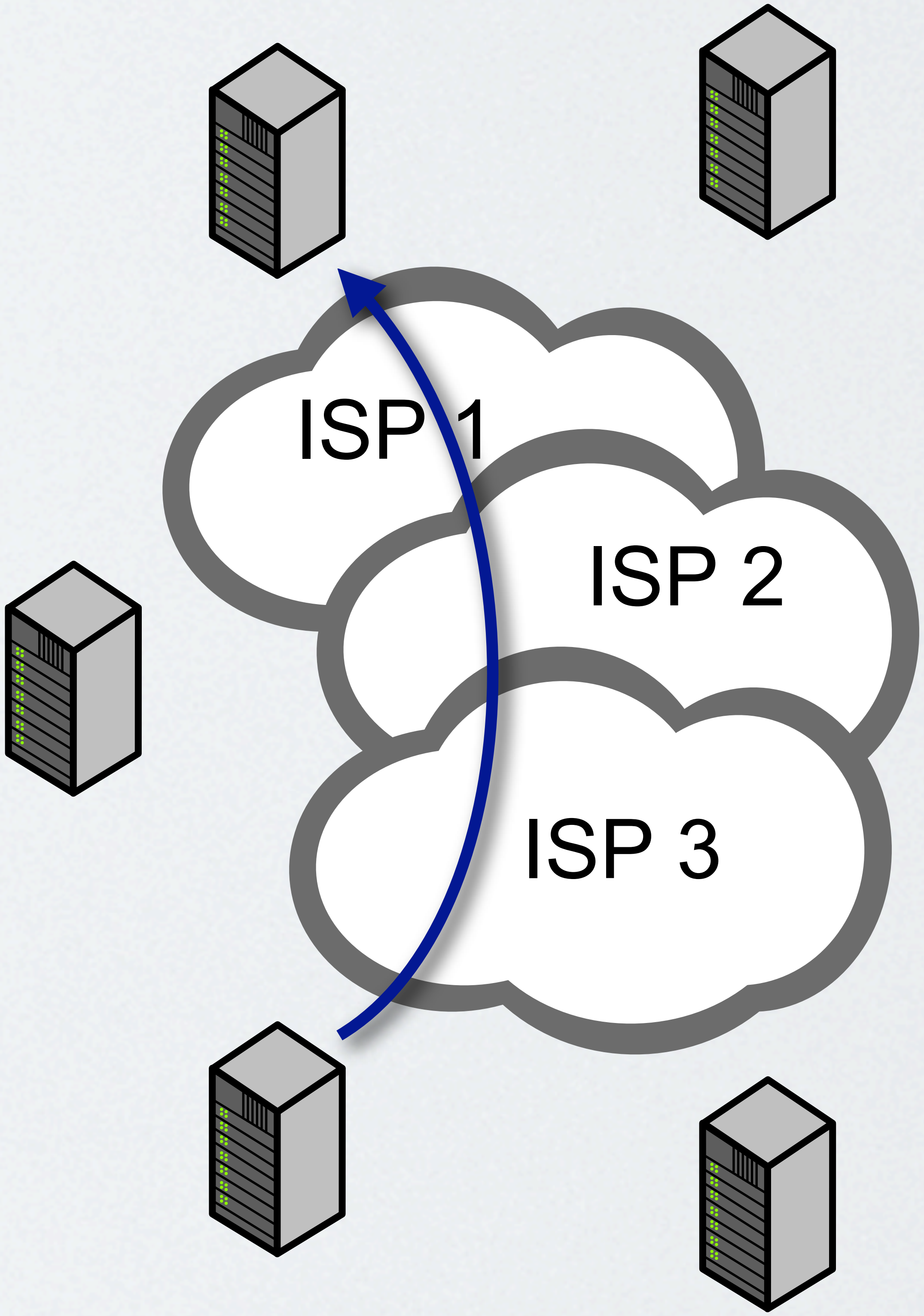


Modeling Overlay Preferences

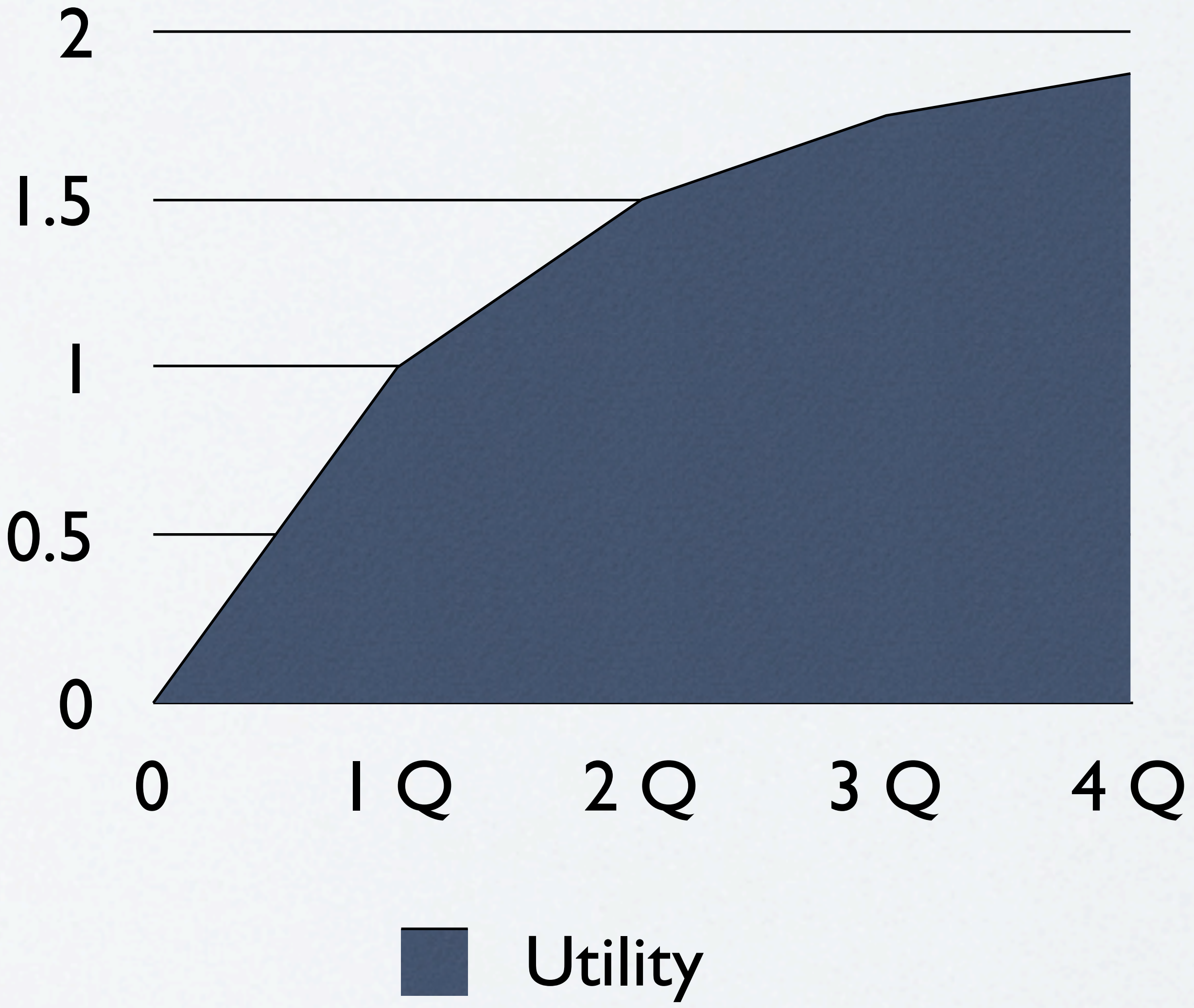
- Diminishing marginal utility on the amount of resources provided by a single site

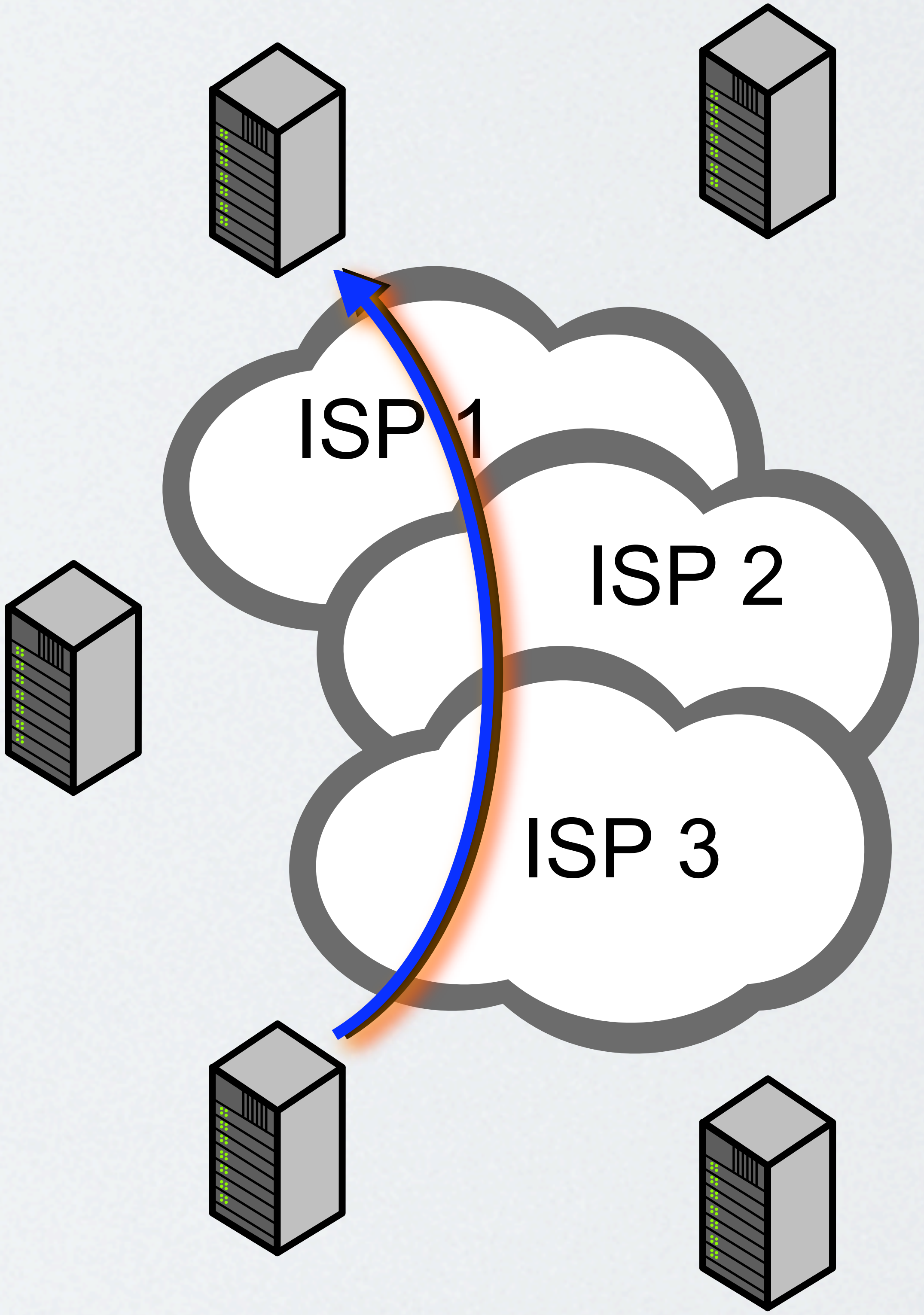




Modeling Overlay Preferences

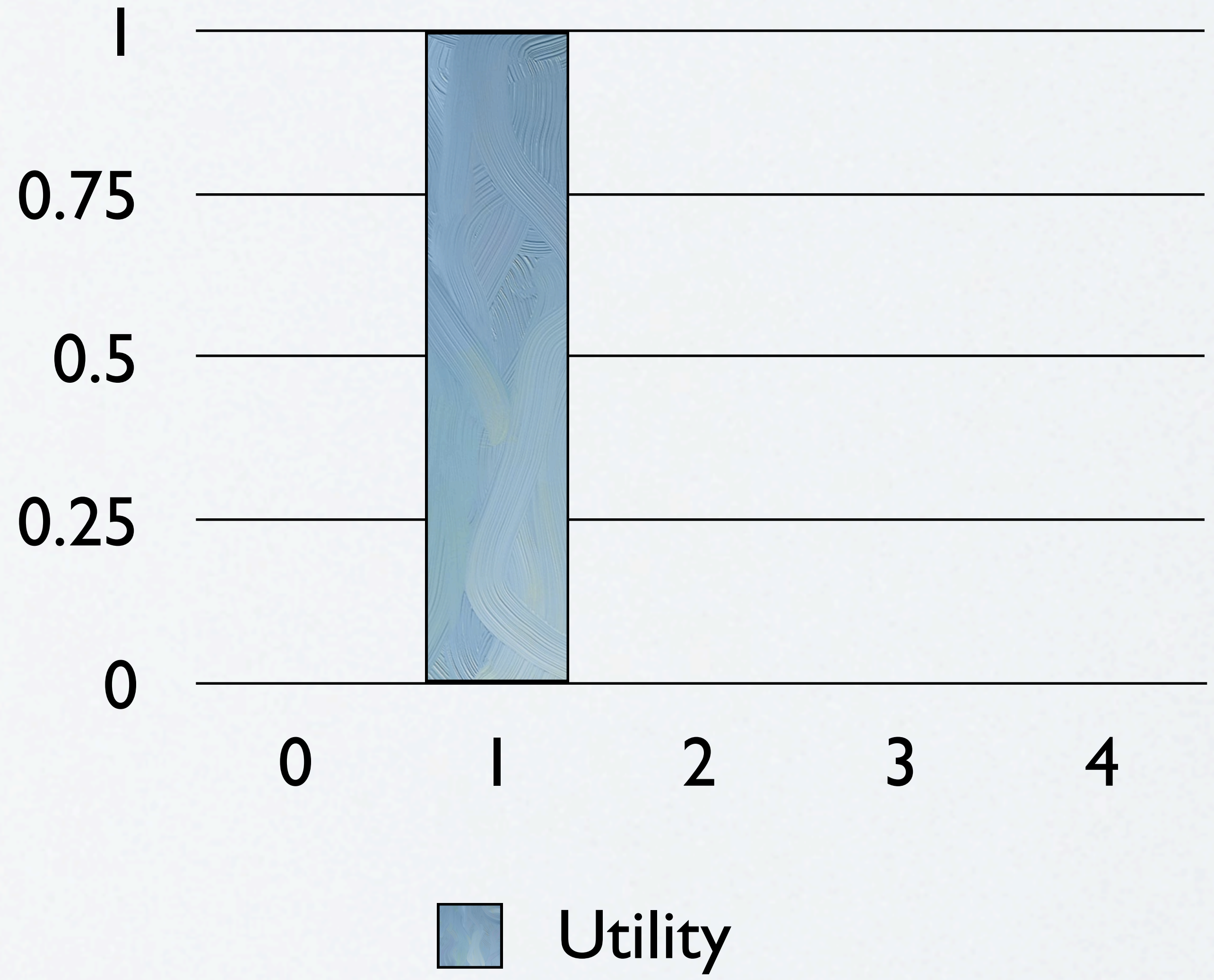
- Diminishing marginal utility on the quality that a given site is able to provide

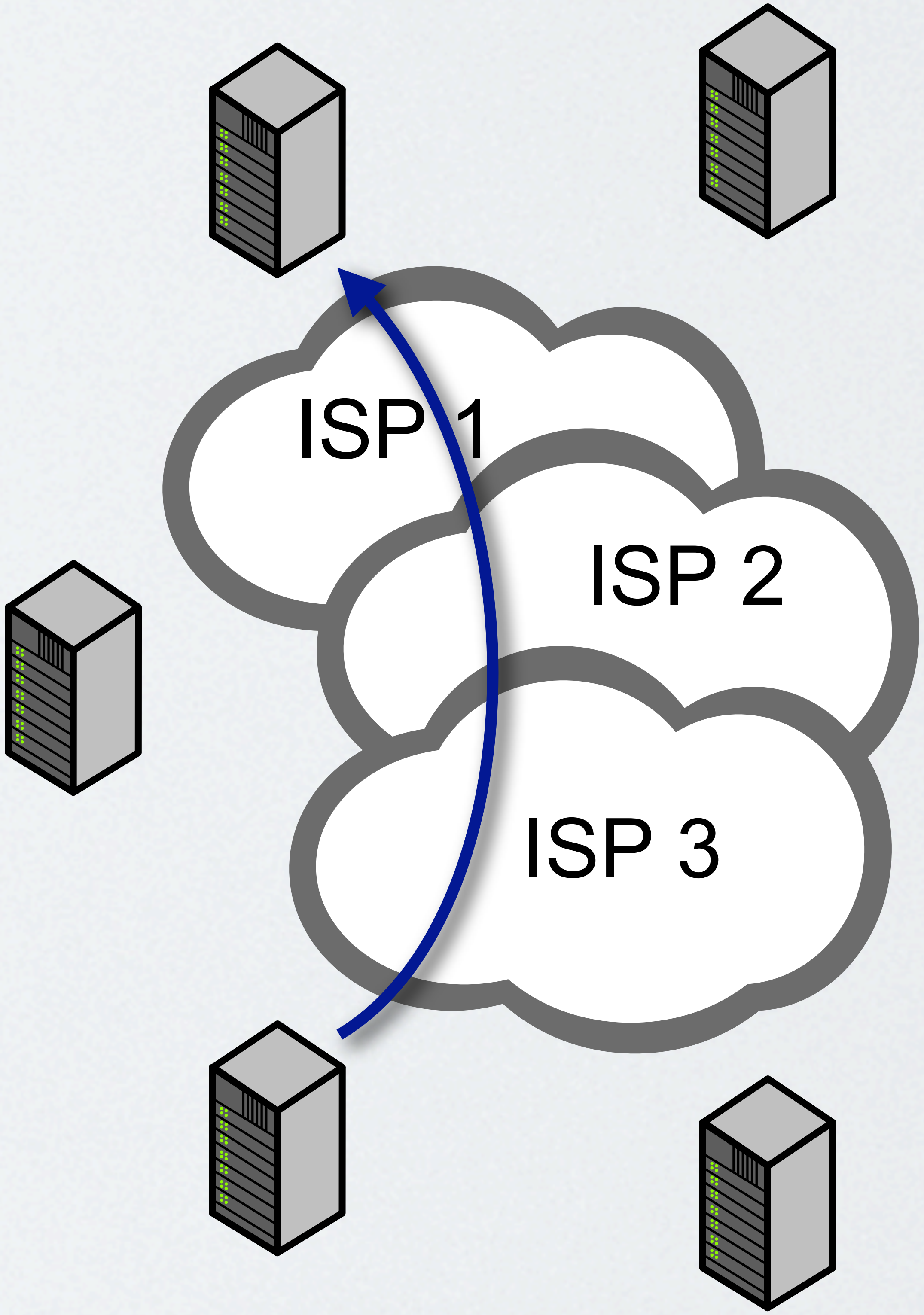




Modeling Overlay Preferences

- Diminishing marginal utility on the number of sites that supply a site with resources





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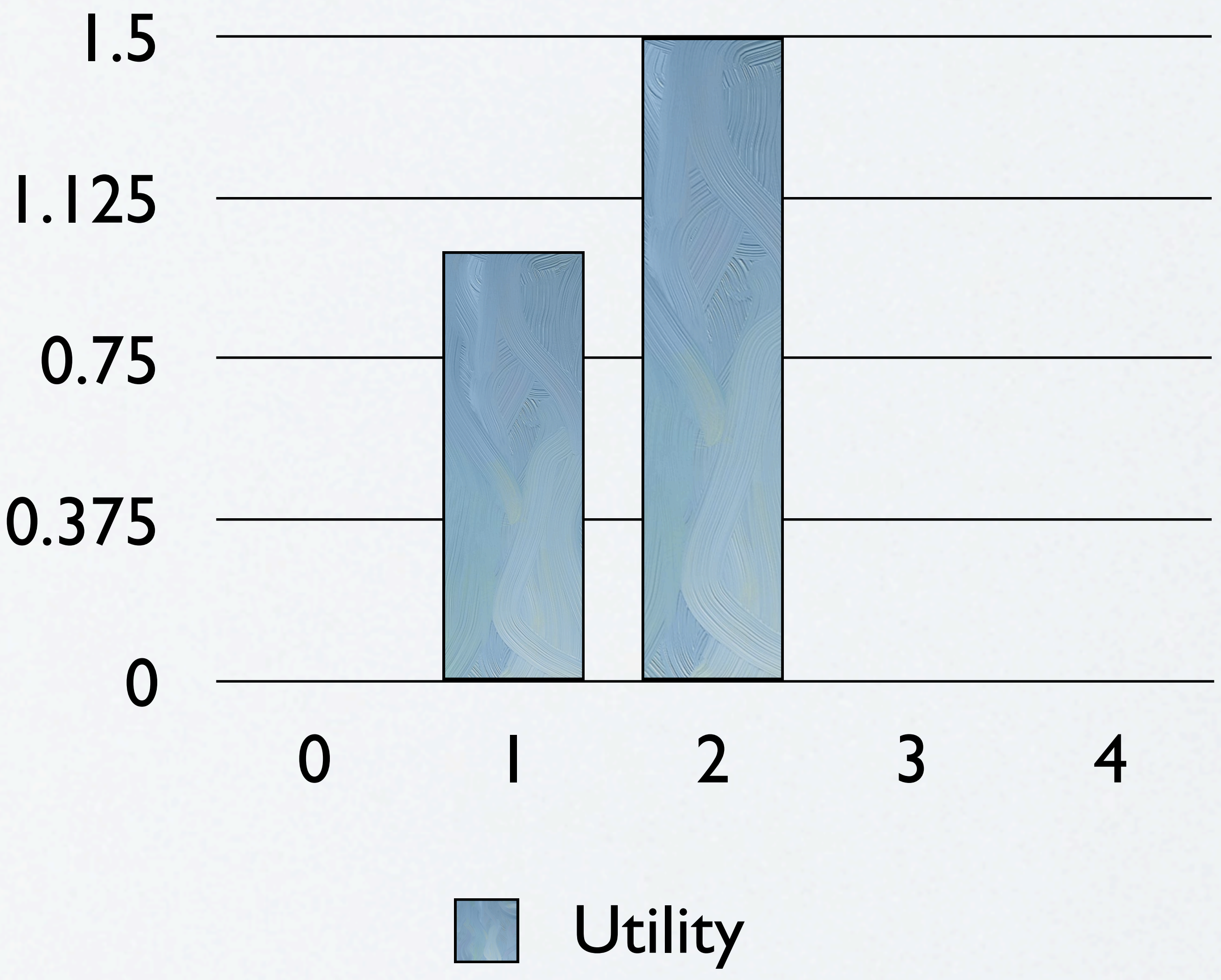

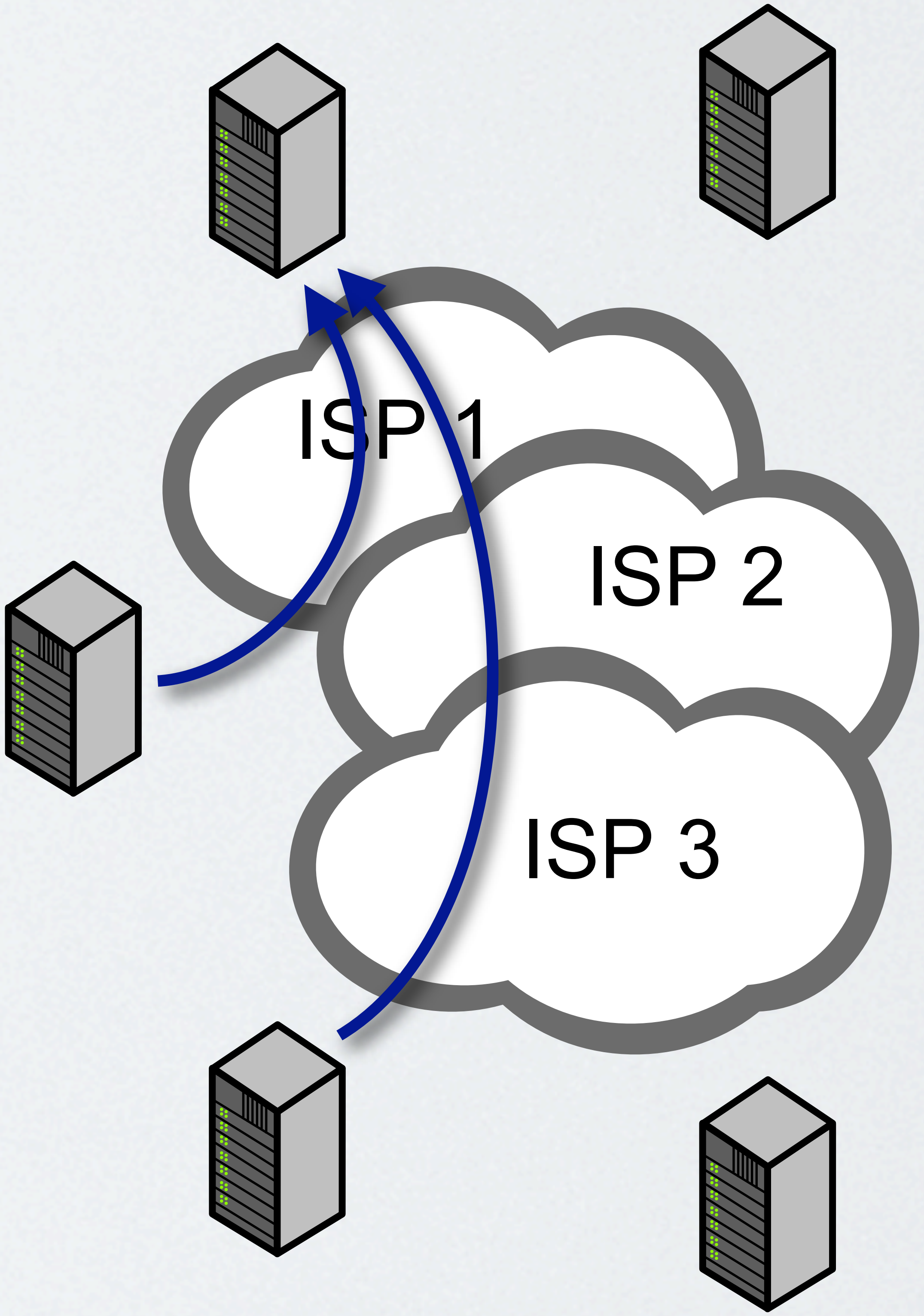

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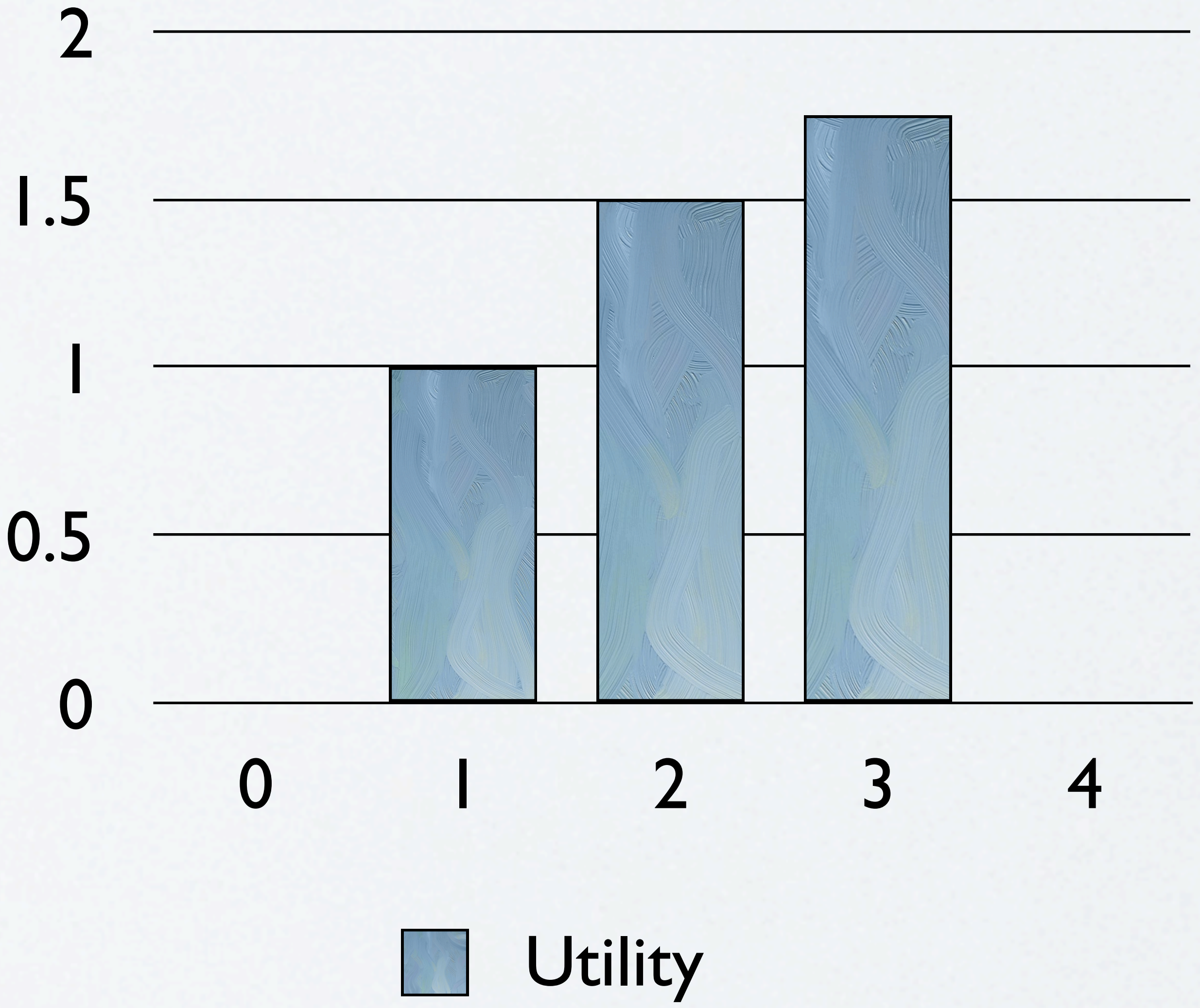

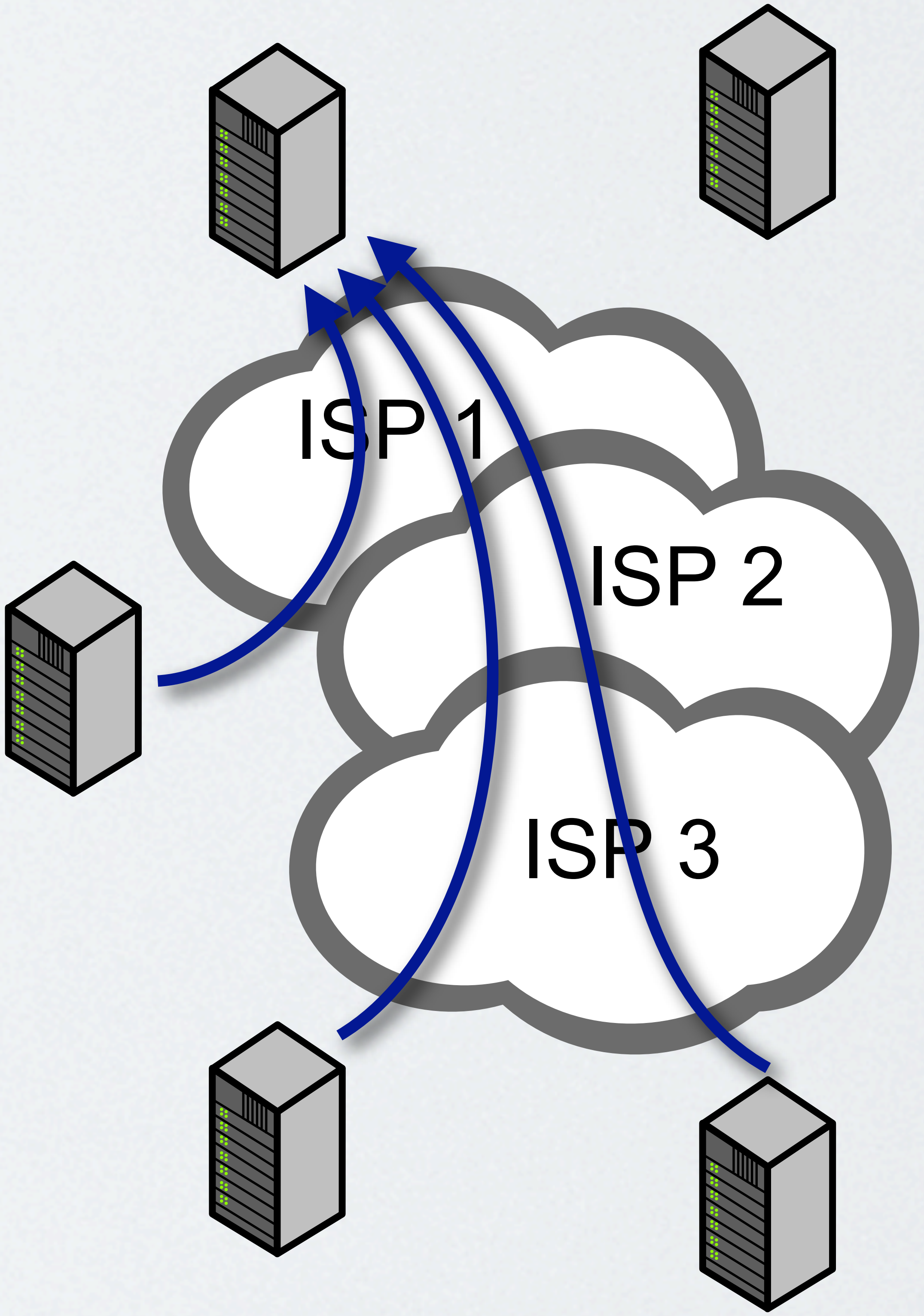

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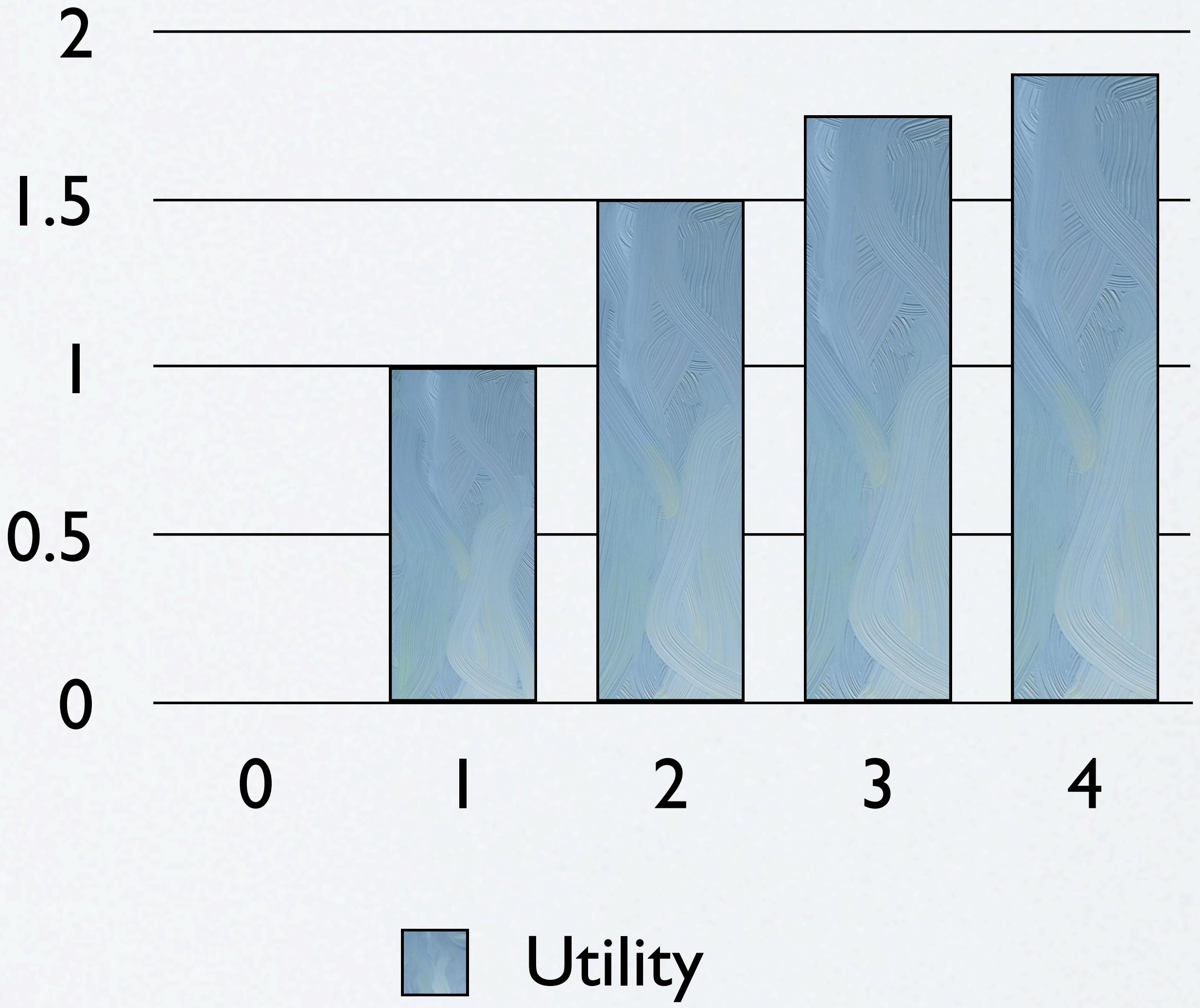
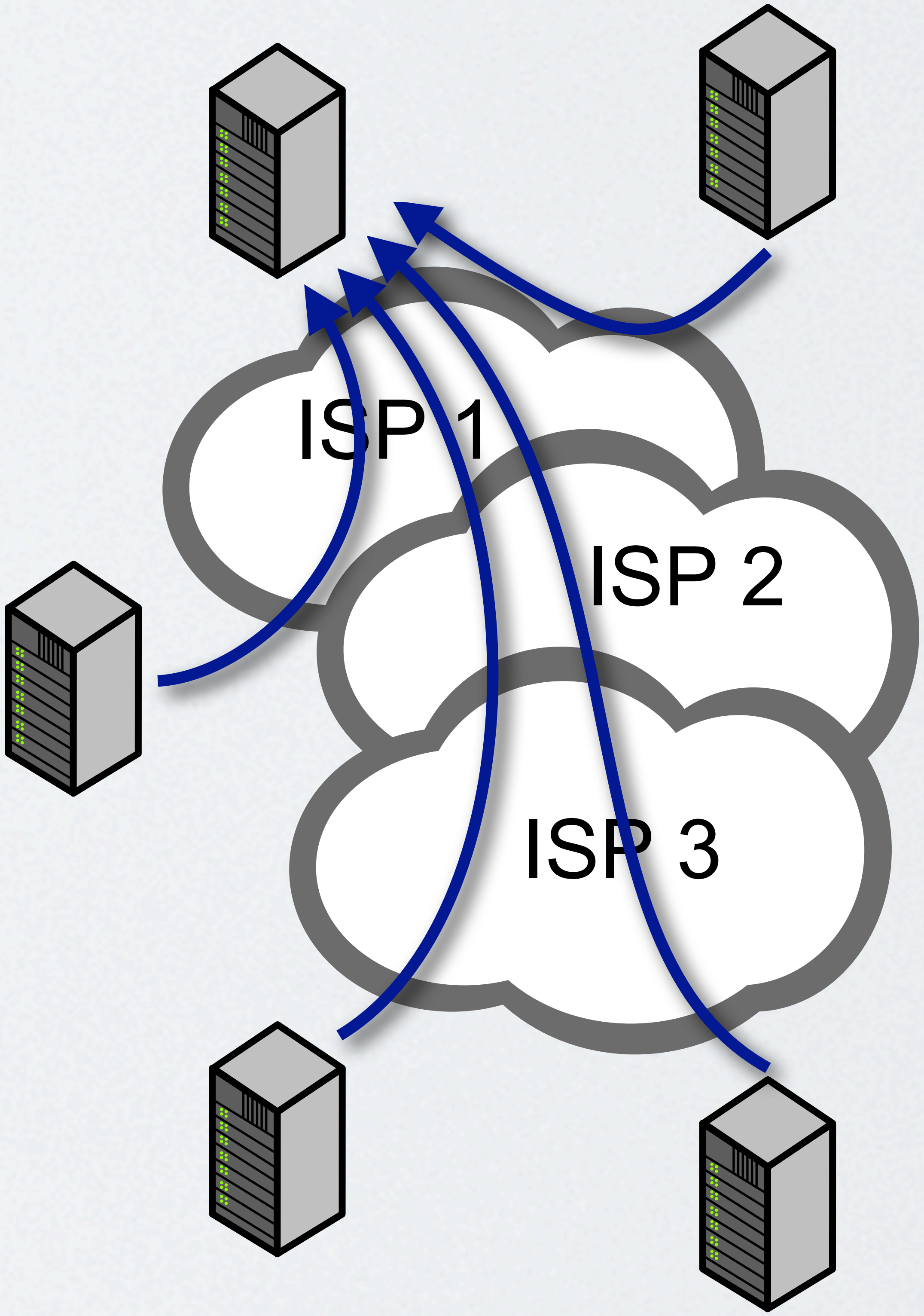




Modeling Overlay Preferences

- We propose an extremely simple, quasilinear utility function for each site that has all these properties:

$$\overbrace{U_i}^{\text{Utility for site } \textbf{\textit{i}}} = \beta_i \overbrace{\left(\underbrace{\sum_{k \in \mathsf{N}, s \in \mathcal{I}}}_{\text{Sum over all sites } \textbf{\textit{k}} \text{ and ISPs } \textbf{\textit{s}}} \underbrace{b_{ski}^{\alpha_i}}_{\text{Amount of resources from site } \textbf{\textit{k}} \text{ over ISP } \textbf{\textit{s}}} \underbrace{q_{ski}^{\gamma_i}}_{\text{Resource quality from site } \textbf{\textit{k}} \text{ over ISP } \textbf{\textit{s}}} \right)^{\delta_i}}^{\text{Total benefit that site } \textbf{\textit{i}} \text{ obtains from resources from all other sites at their given quality}} - \overbrace{\sum_{k \in \mathsf{N}, s \in \mathcal{I}} \underbrace{p_{ski}}_{\text{Price for resources from site } \textbf{\textit{k}} \text{ over ISP } \textbf{\textit{s}}} b_{ski}}^{\text{Total cost that site } \textbf{\textit{i}} \text{ is required to pay to obtain for resources from all other sites}}$$

Modeling Overlay Preferences

- We propose an extremely simple, quasilinear utility function for each site that has all these properties:

$$U_i = \overset{\text{Increasing}}{\overline{\beta_i}} \overset{\text{Increasing with diminishing returns}}{\overline{\left(\sum_{k \in \mathsf{N}, s \in \mathcal{I}} \underset{\text{Increasing with diminishing returns}}{\underline{b_{ski}^{\alpha_i}}} \underset{\text{Increasing with diminishing returns}}{\underline{q_{ski}^{\gamma_i}}} \right)^{\delta_i}}} - \overset{\text{Increasing}}{\overline{\sum_{k \in \mathsf{N}, s \in \mathcal{I}} p_{ski} b_{ski}}}$$

$$\boxed{0 \leq \{\alpha_i, \gamma_i, \delta_i\} \leq 1}$$

Modeling Overlay Preferences

- We thus formulate the following optimisation problem:

Sum of over all sites ***i***

$$\text{Maximise:} \quad U = \overline{\sum_{i \in \mathsf{N}}} U_i$$

$$U = \sum_{i \in \mathsf{N}} \left(\beta_i \left(\sum_{k \in \mathsf{N}, s \in \mathcal{I}} b_{ski}^{\alpha_i} q_{ski}^{\gamma_i} \right)^{\delta_i} - \sum_{k \in \mathsf{N}, s \in \mathcal{I}} p_{ski} b_{ski} \right)$$

Modeling Overlay Preferences

- The unconstrained solution to this problem is:

$$b_{sji} = \arg\max U$$

$$b_{sji} = (\beta_i \alpha_i \delta_i)^{\frac{1}{1-\alpha_i \delta_i}} \frac{\left(\frac{q_{sji}^{\gamma_i}}{p_{sji}}\right)^{\frac{1}{1-\alpha_i}}}{\left(\sum_{k \in \mathsf{N}, t \in \mathcal{I}} p_{tki} \left(\frac{q_{tki}^{\gamma_i}}{p_{tki}}\right)^{\frac{1}{1-\alpha_i}}\right)^{\frac{1-\delta_i}{1-\alpha_i \delta_i}}}$$

- Not unexpectedly, b_{sji} is a function of:
  - the utility parameters $\alpha_i, \beta_i, \delta_i, \gamma_i$
  - the overlay link prices p_{tki}
  - the overlay cost-benefit ratios $\frac{q_{tki}^{\gamma_i}}{p_{tki}}$

Modeling Overlay Preferences

- We can extend this solution by considering a ***budget constraint***:

$$\text{Maximise:} \quad U = \sum_{i \in \mathsf{N}} U_i$$

$$U = \sum_{i \in \mathsf{N}} \left(\beta_i \left(\sum_{k \in \mathsf{N}, s \in \mathcal{I}} b_{ski}^{\alpha_i} q_{ski}^{\gamma_i} \right)^{\delta_i} - \sum_{k \in \mathsf{N}, s \in \mathcal{I}} p_{ski} b_{ski} \right)$$

$$\text{Subject to:} \quad \sum_{\underline{i \in \mathsf{N}, k \in \mathsf{N}, t \in \mathcal{I}}} b_{ski} p_{ski} \leq \mathcal{B}$$

Sum over all origin/destination
site pairs and over all ISPs

Modeling Overlay Preferences

- The constrained solution to this problem is:

$$b_{sji} = (\beta_i \alpha_i \delta_i)^{\frac{1}{1-\alpha_i \delta_i}} \left(\frac{1}{1+\lambda} \right)^{\frac{1}{1-\alpha_i}} \frac{\left(\frac{q_{sji}^{\gamma_i}}{p_{sji}} \right)^{\frac{1}{1-\alpha_i}}}{\left(\sum_{k \in \mathrm{N}, t \in \mathcal{I}} p_{tki} \left(\frac{q_{tki}^{\gamma_i}}{p_{tki}} \right)^{\frac{1}{1-\alpha_i}} \right)^{\frac{1-\delta_i}{1-\alpha_i \delta_i}}}$$

- Where λ is a Lagrange multiplier. Of course, this simplifies to the unbounded case if $\lambda = 0$ (constraint does not bind).
- To find λ, we define $\hat{\mathcal{B}}_i$, the total flow cost for site ***i*** *had the budget condition not been binding:*

$$\hat{\mathcal{B}}_i = \sum_{k \in \mathrm{N}, s \in \mathcal{I}} p_{ski} \hat{b}_{ski}$$

Modeling Overlay Preferences

- Then, λ can be found by solving the following equation:

$$\sum_{i \in \mathsf{N}} \left(\frac{1}{1+\lambda} \right)^{\frac{1}{1-\alpha_i}} \hat{\mathcal{B}}_i = \mathcal{B}$$

- Simple procedure for constrained problem:

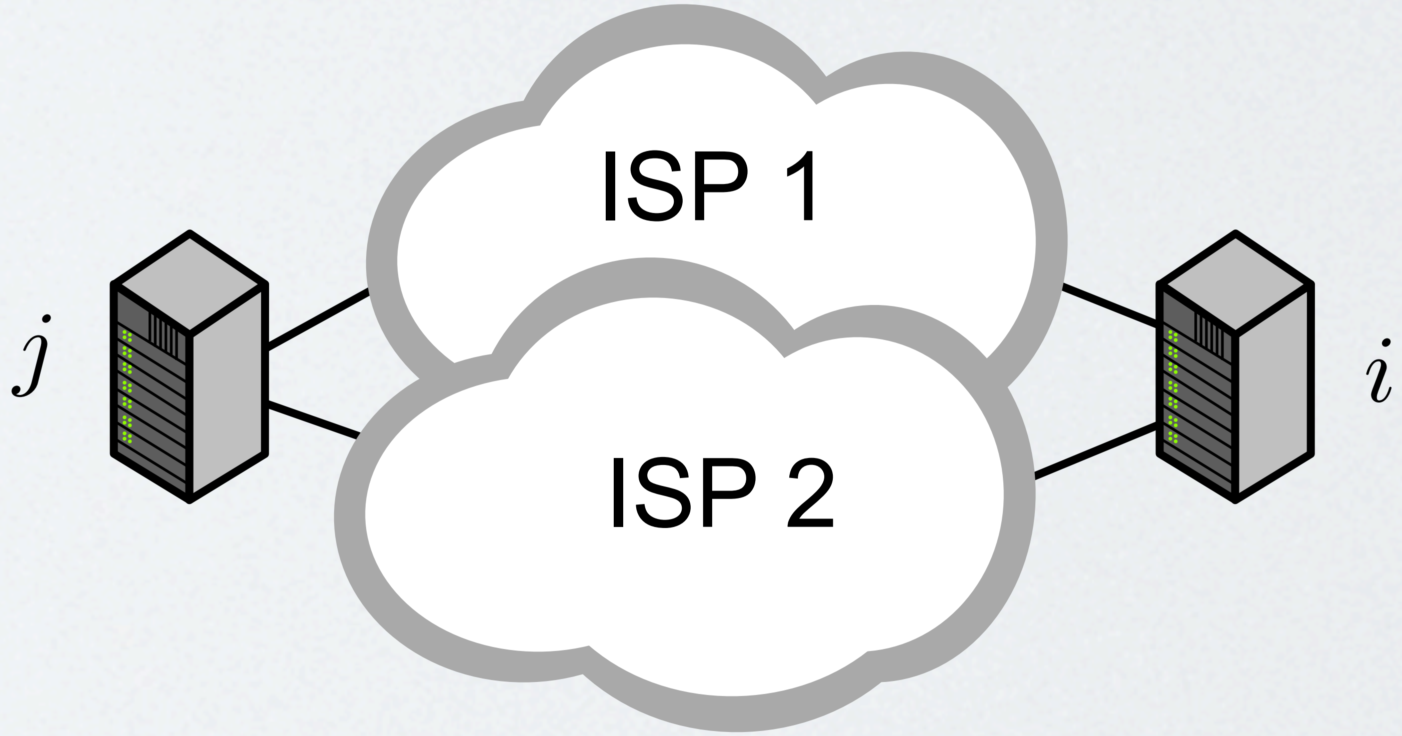
- Calculate traffic matrix ignoring binding constraint
- Calculate $\hat{\mathcal{B}}_i$ using this traffic matrix

- If $\mathcal{B} \geq \sum_{i \in \mathsf{N}} \hat{\mathcal{B}}_i$, the budget condition does not bind and $\lambda = 0$

- Else, find λ and obtain correct traffic matrix b_{sji}

Modeling Overlay Preferences (Example)

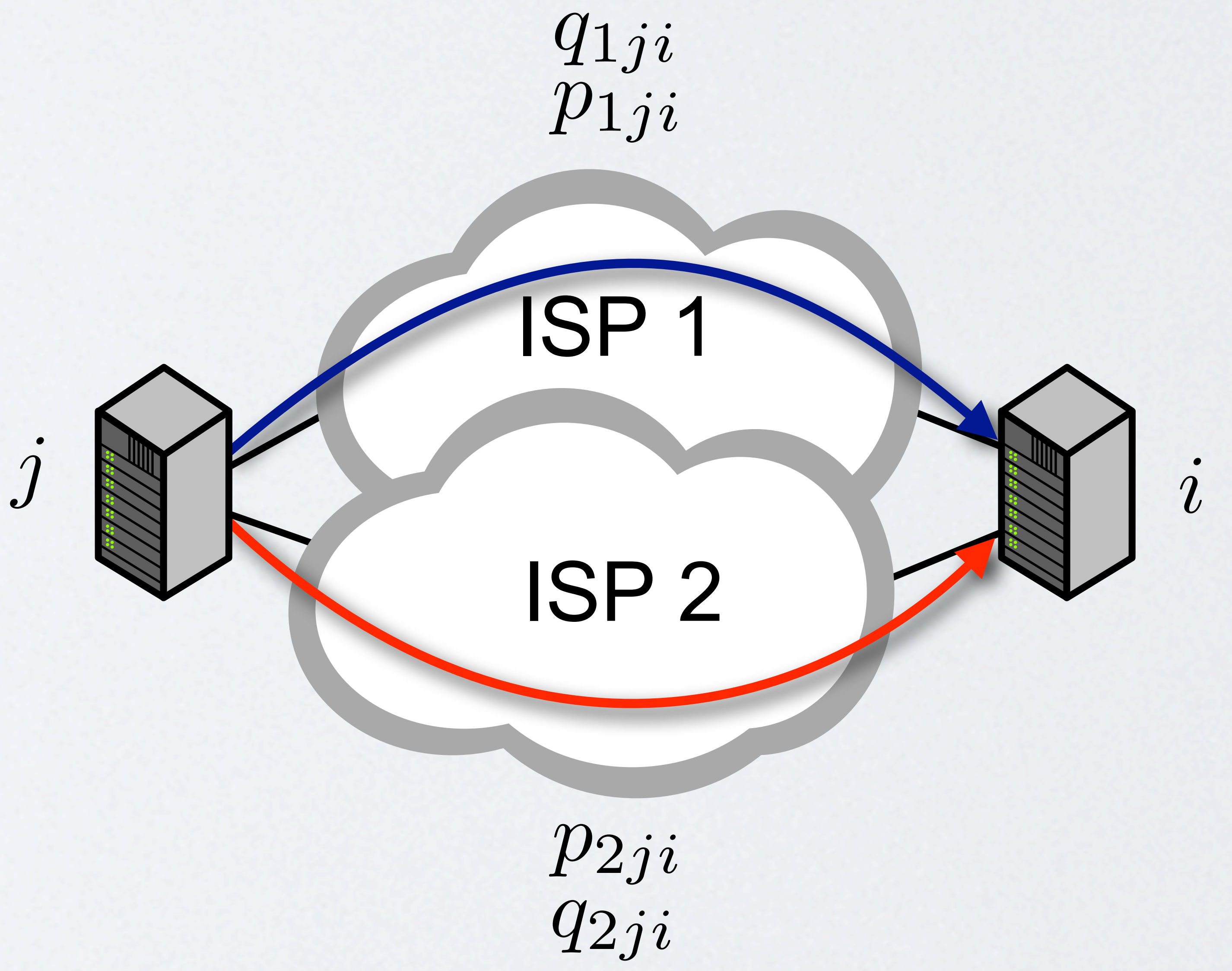
- Consider two overlay sites, ***i*** and ***j***, that can reach each other through two given ISPs



Modeling Overlay Preferences (Example)

- Consider two overlay sites, ***i*** and ***j***, that can reach each other through two given ISPs

- We analyse the allocation of flow volumes to ISPs, the total cost and the total utility as the price p_{1ji} changes.



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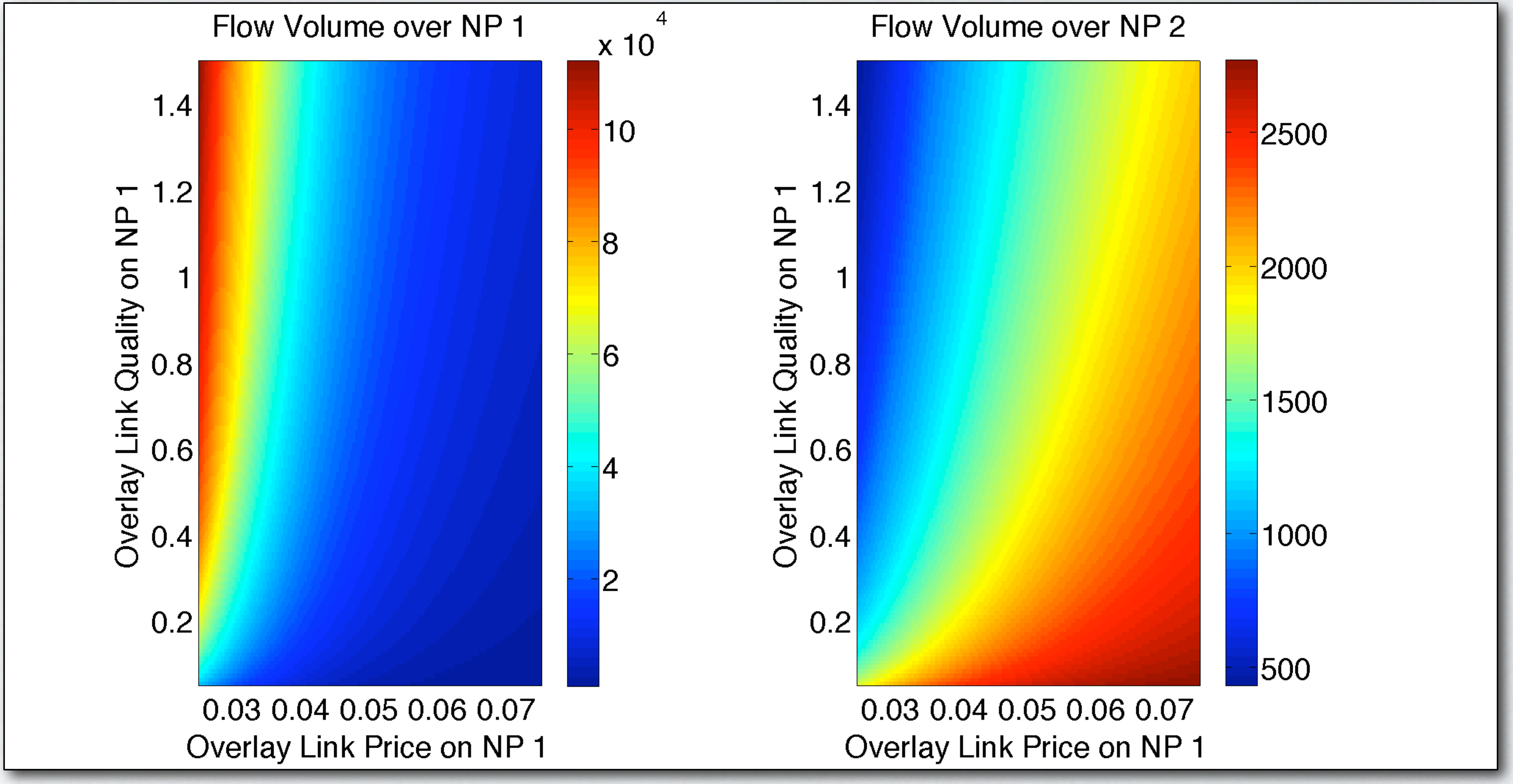
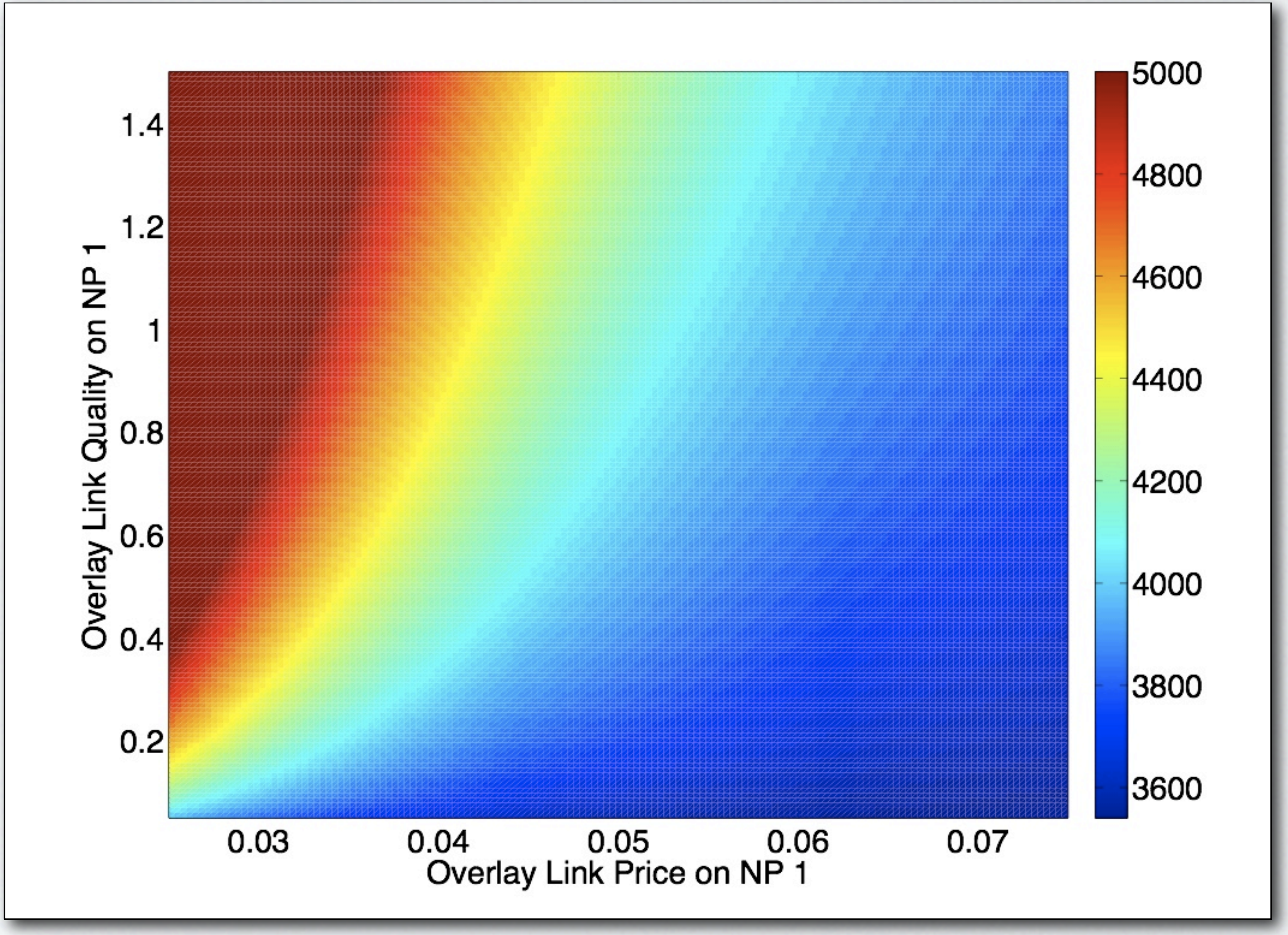


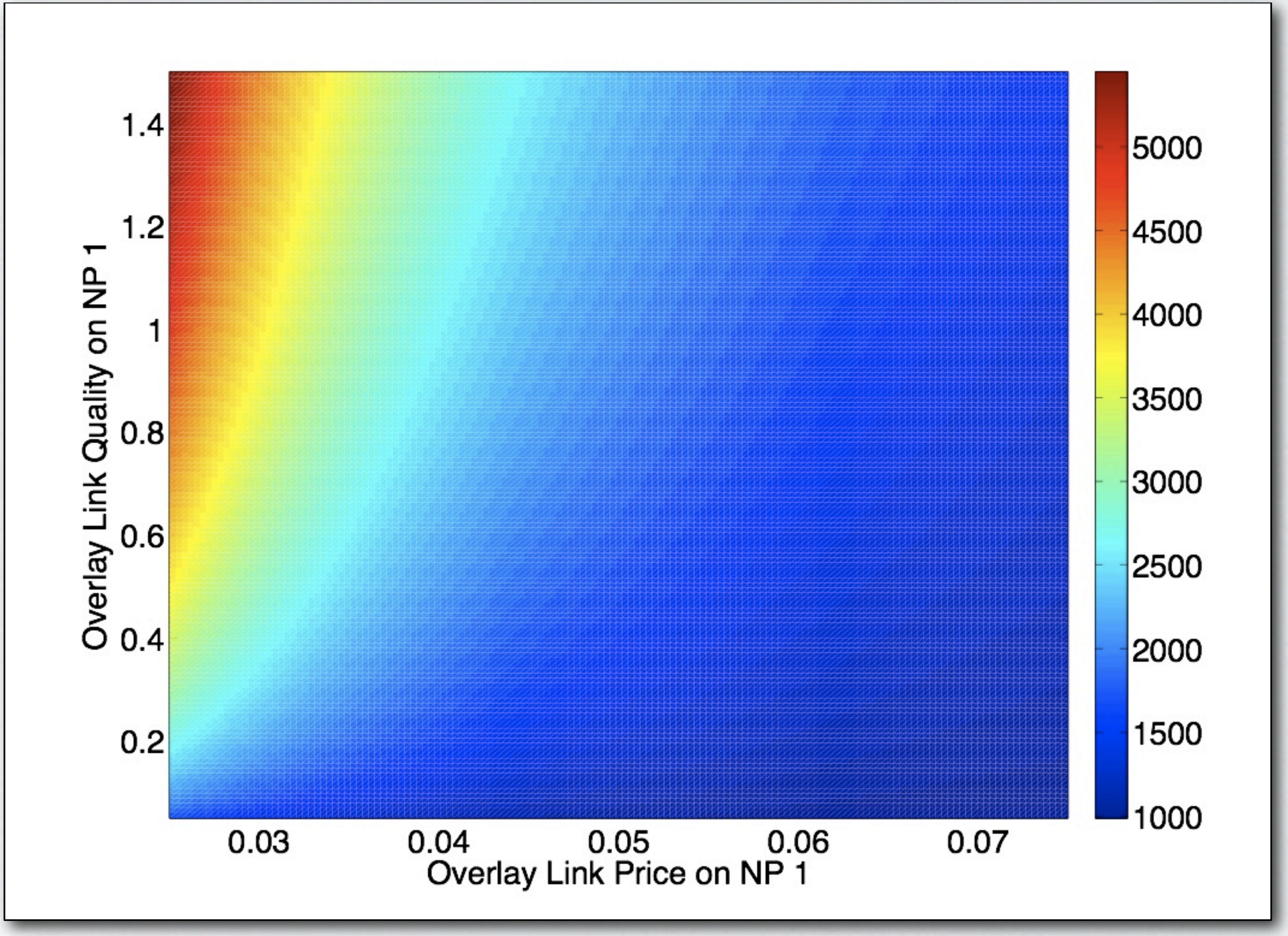
Modeling Overlay Preferences (Example)

Total ESP Cost



Modeling Overlay Preferences (Example)

Total ESP Utility



Fitting Aggregate Overlay Preferences

- To be used in practice, the model presented requires the estimation of $\alpha_i, \beta_i, \gamma_i, \delta_i$ and both q_{ski} and p_{ski}.

- Furthermore, it may be of interest for a given ISP to model the ***demand aggregate*** provided by all of its ESPs, rather than the preferences of each single ESP

- The obvious data-driven approach for this is through regression. If we denote the flow volume of origin-destination site pair k with B_k, we seek an approximate such that $B_k = f(p_1, p_2, \ldots, p_k, \ldots) \quad \forall k$

Fitting Overlay Preferences

- We propose to use the well known Cobb-Douglas function for this demand model:

$$\log B_k = \eta_0^k + \sum_{\xi \in \mathsf{L}} \eta_\xi^k \log p_\xi$$

- Thus, we explicitly model the *price elasticity of demand* η_k^k and the *cross elasticity of demand* η_ξ^k:

$$\frac{\partial \log B_k}{\partial \log p_\xi} = \eta_\xi^k$$

- This allows the modeling of ***flow substitution*** effects

Fitting Overlay Preferences

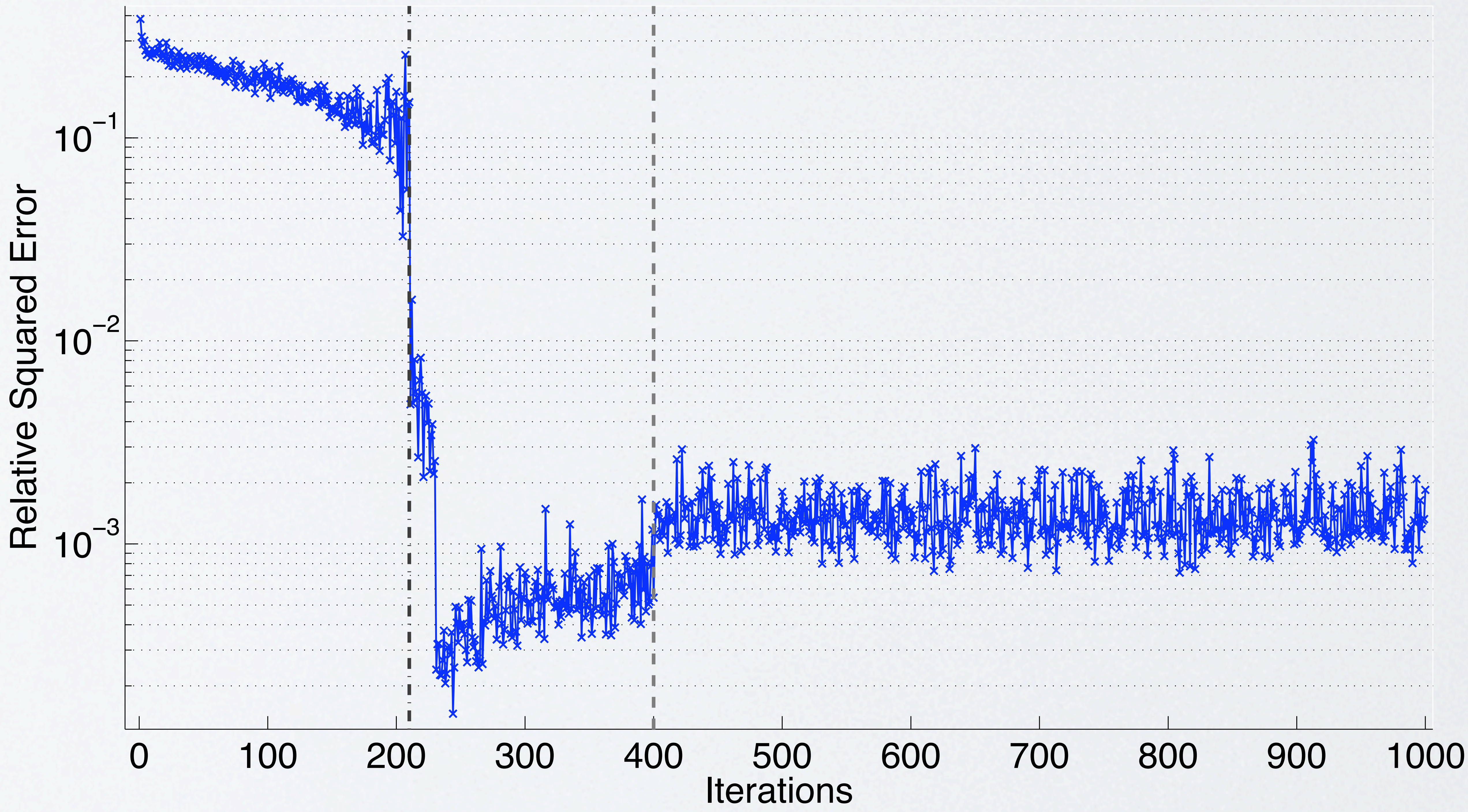
Range	**Category**	**Responsiveness**	**Change in demand for k given that ξ increases in price**
$\eta_\xi^k \rightarrow -\infty$	Complement	Perfectly Elastic	Arbitrary Decrease
$-\infty < \eta_\xi^k < -1$	Complement	Elastic	Large Decrease
$\eta_\xi^k = -1$	Complement	Unitary Elastic	Comparable Decrease
$-1 < \eta_\xi^k < 0$	Complement	Inelastic	Small Decrease
$\eta_\xi^k = 0$	Independent	Perfectly Inelastic	No Change
$0 < \eta_\xi^k < 1$	Substitute	Inelastic	Small Increase
$\eta_\xi^k = 1$	Substitute	Unitary Elastic	Comparable Increase
$1 < \eta_\xi^k < \infty$	Substitute	Elastic	Large Increase
$\eta_\xi^k \rightarrow \infty$	Substitute	Perfectly Elastic	Arbitrary Increase

Testing the Overlay Preference Fitting Procedure

- Fitting is performed through conventional least-squares regression

- To test the model:
  - Assume single underlying ISP and 15 overlay sites
  - A set of 15 ESPs is created, along with a vector of IID parameters ($\alpha_i, \beta_i, \gamma_i, \delta_i$) for each one.
  - An overlay link quality matrix q_{jk} is generated
  - 400 price vectors are generated, and the response from the aggregate overlay estimated through regression
  - 600 additional price vectors are tested without further update to estimated elasticities

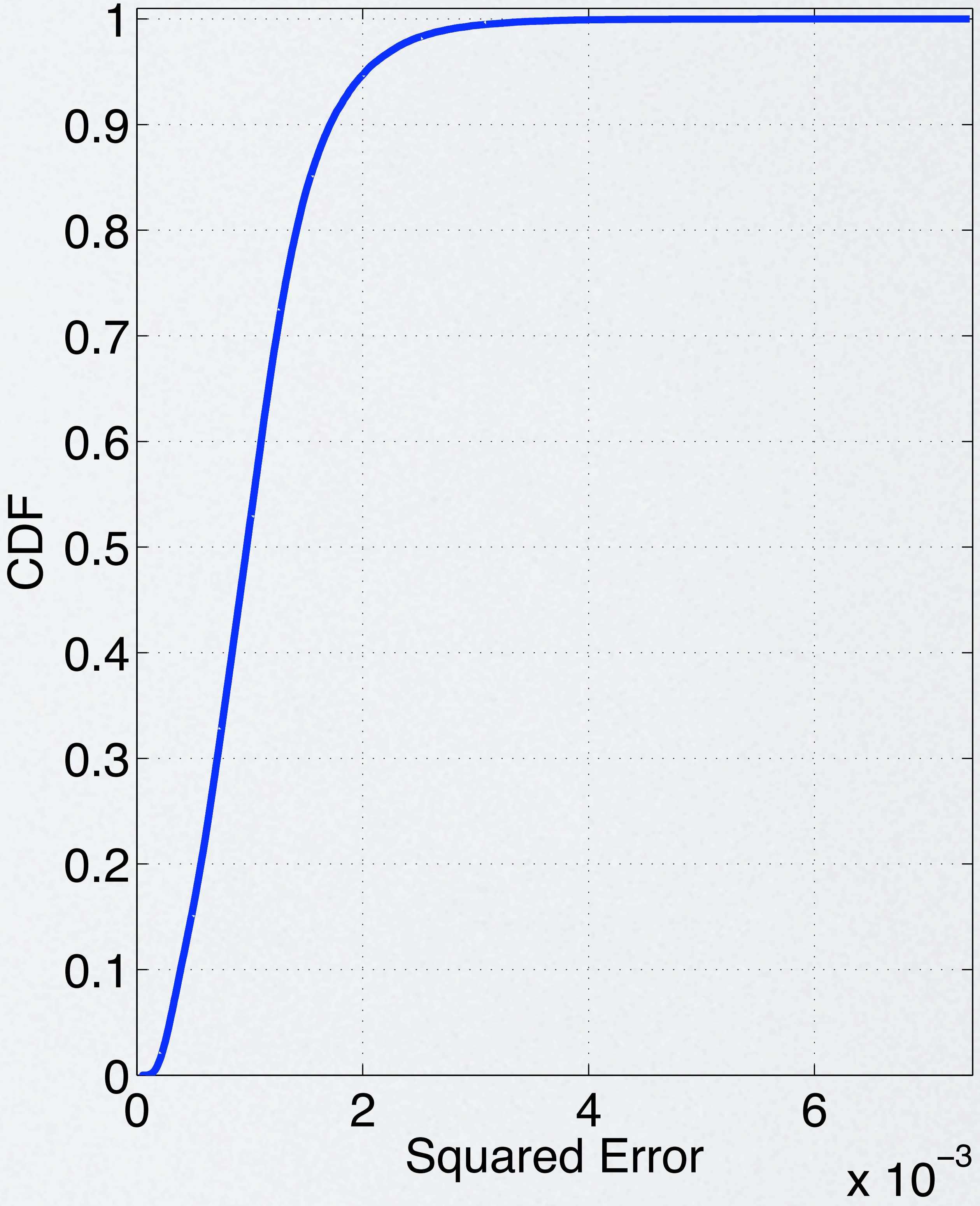
Testing the Overlay Preference Fitting Procedure



Testing the Overlay Preference Fitting Procedure

- Estimation has good relative squared error performance

$$E_{rel} = \frac{||\hat{B} - B||_2}{||B||_2}$$



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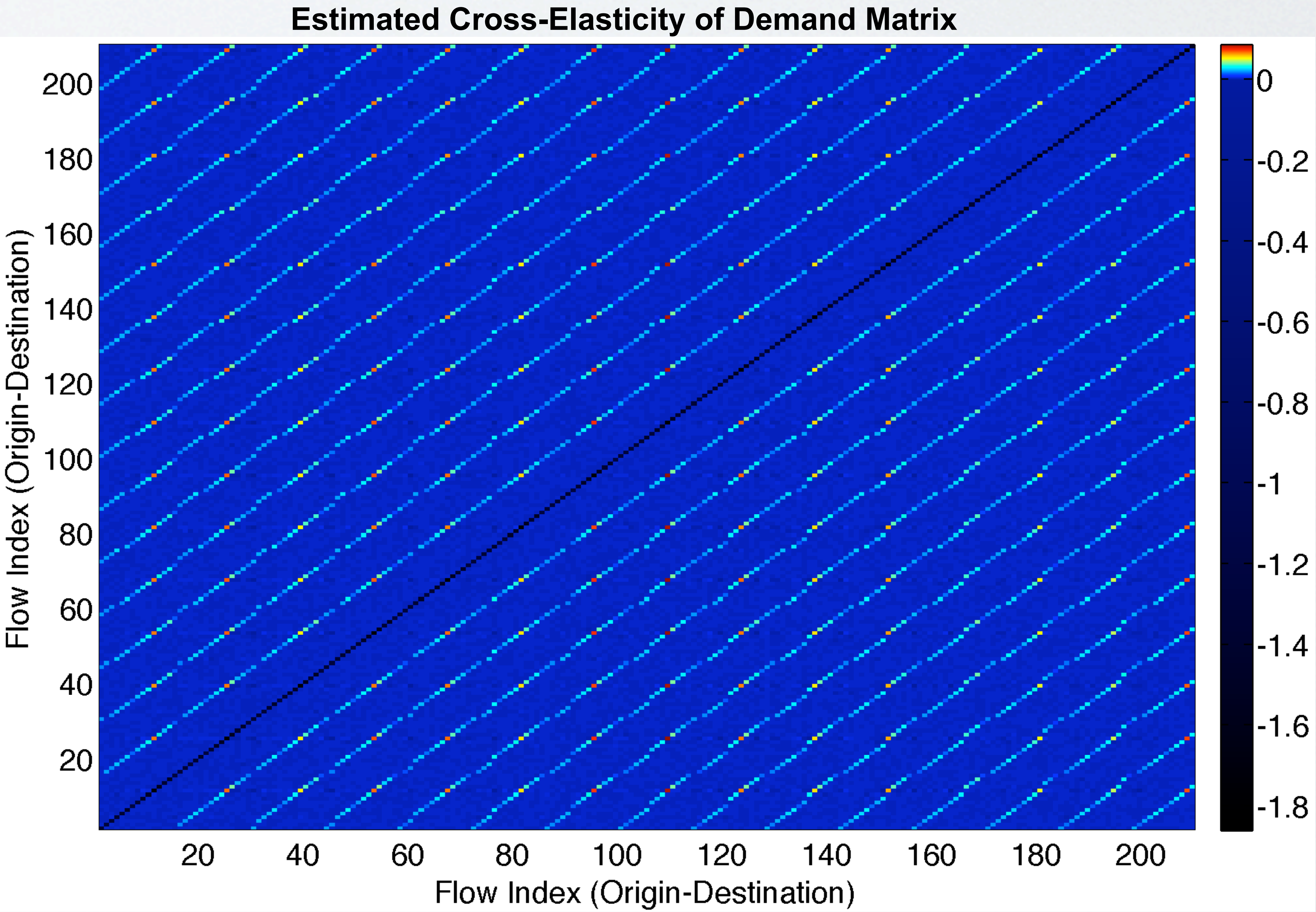
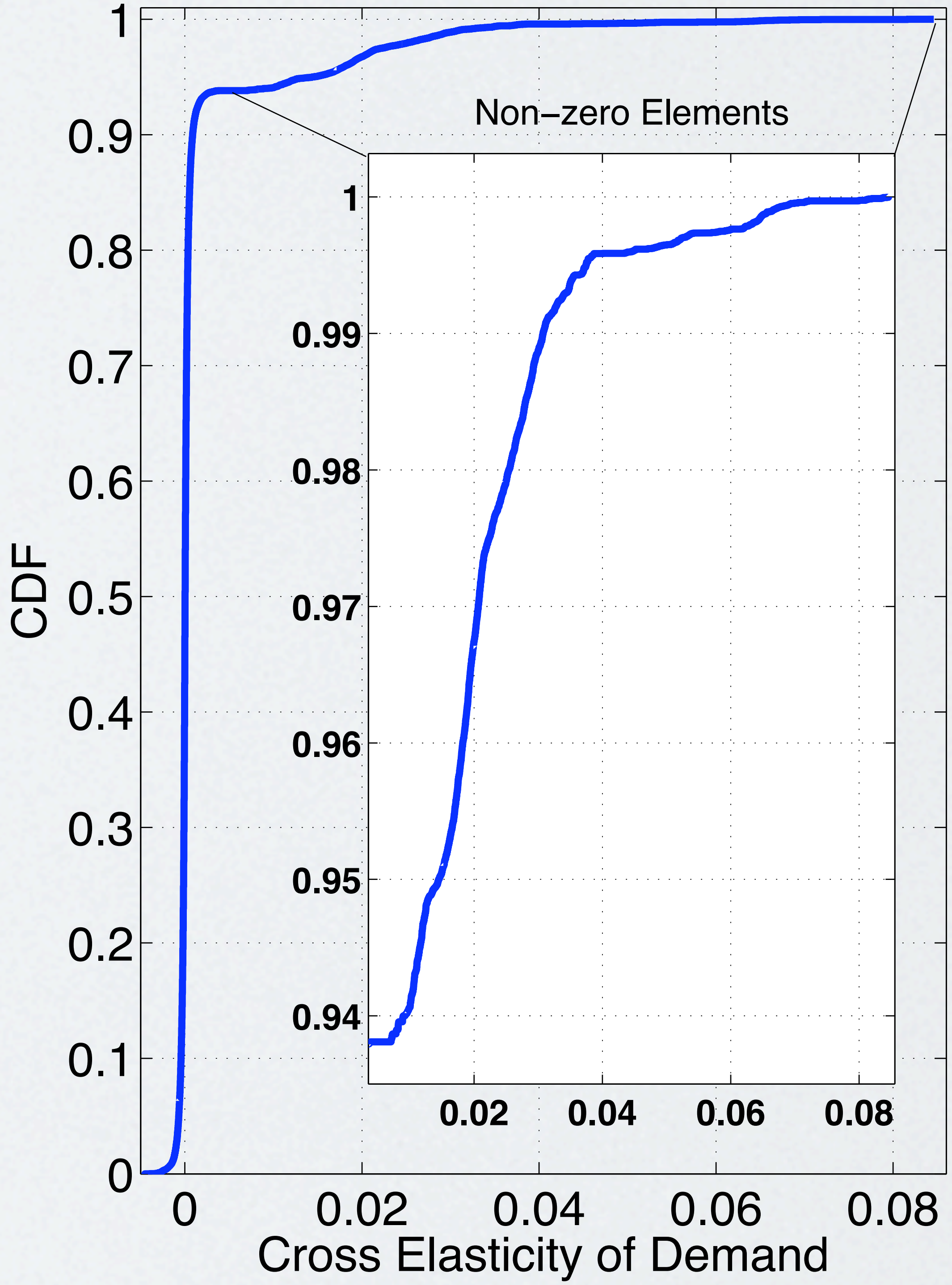


Testing the Overlay Preference Fitting Procedure

Estimated Cross-Elasticity of Demand Matrix

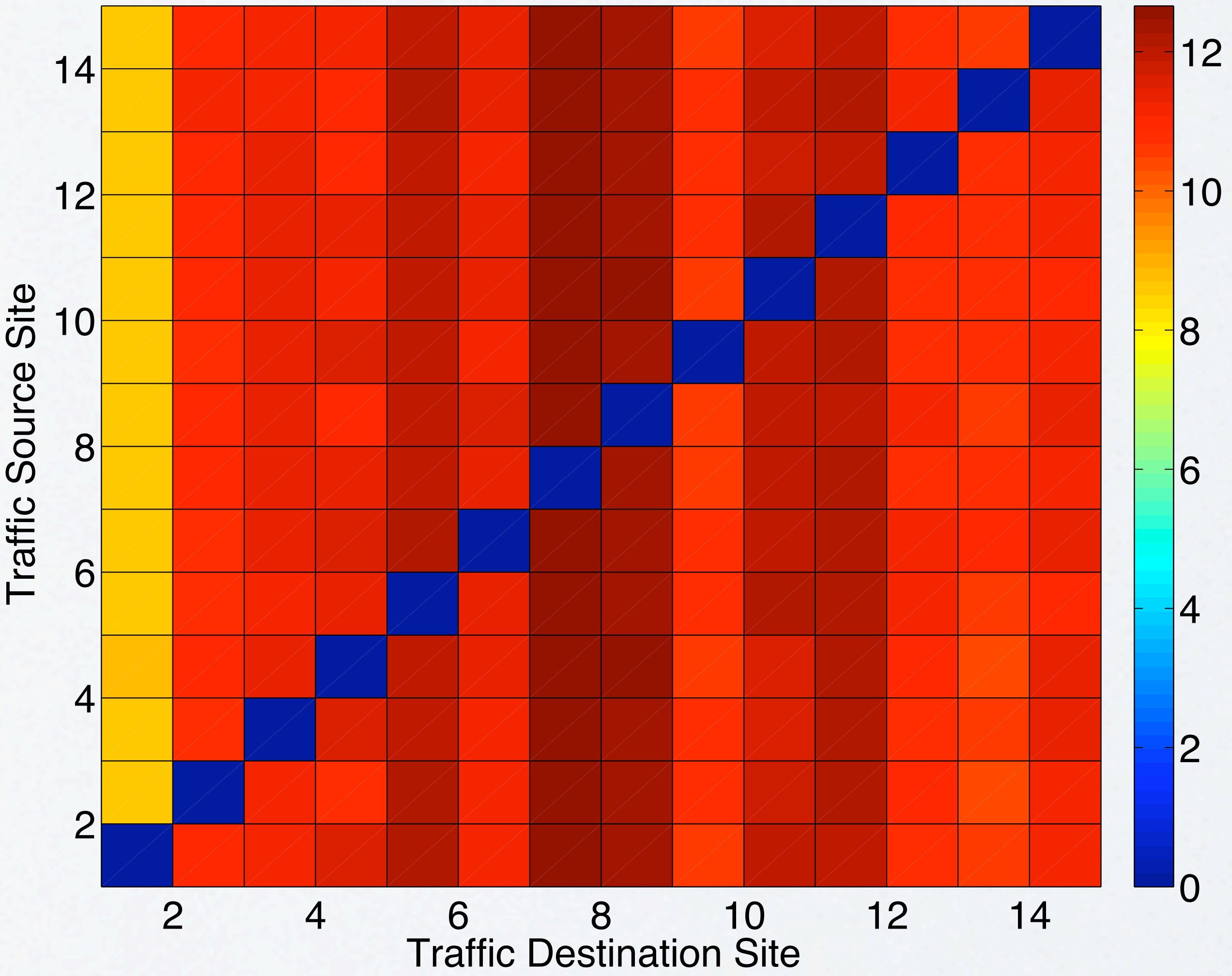
- Cross-elasticites of demand are either
  - ***zero*** - flows are perfectly inelastic, independent products
  - ***positive*** - flows are inelastic, substitute products

- This happens because quality between flows is uncorrelated



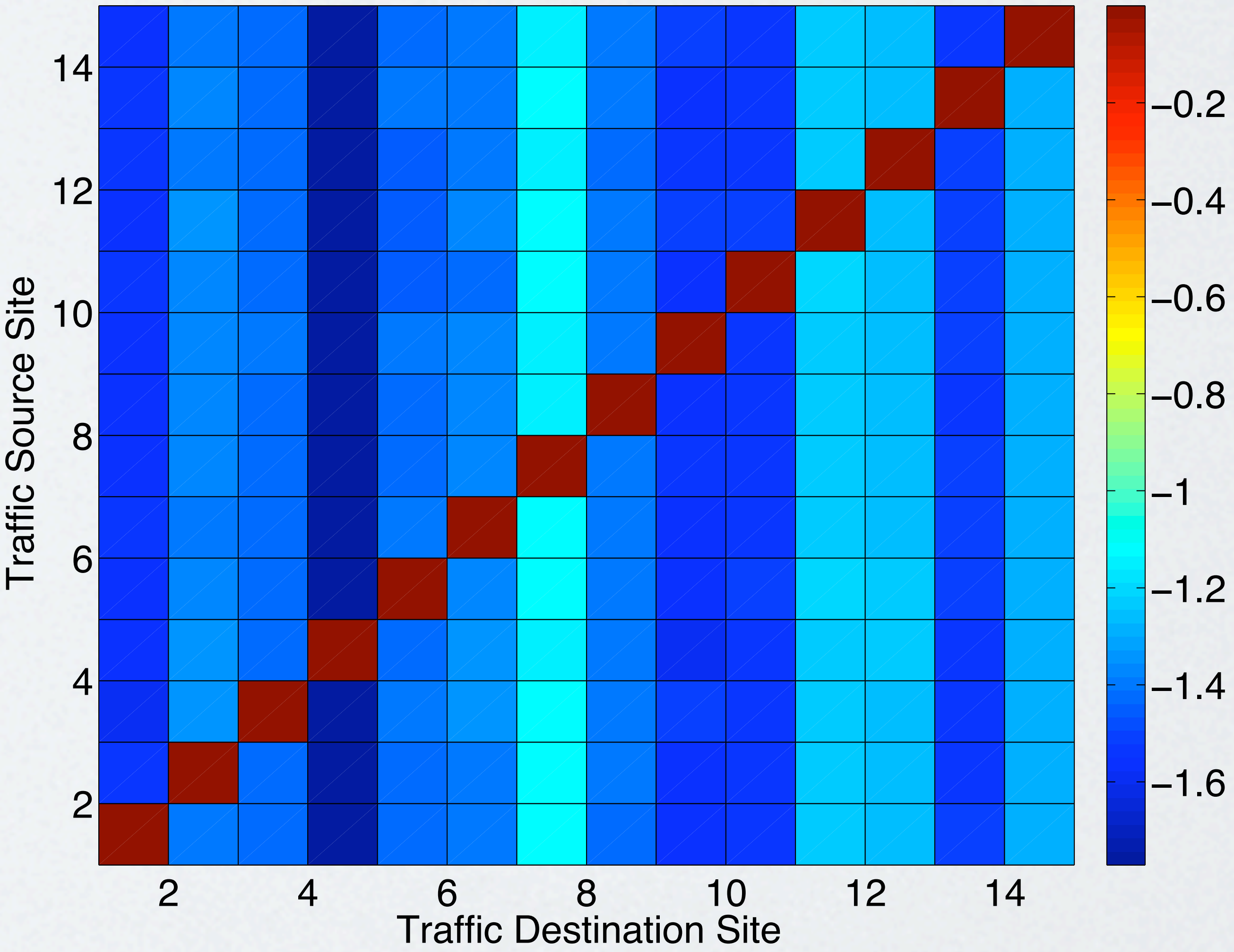
Testing the Overlay Preference Fitting Procedure

$$\eta_k^0$$



Gives indication for demand at unit price

$$-\infty < \eta_k^k < -1$$



Flow demand is elastic with price

Future Work

- How close to reality are these models?
  - We need ***data***

- In the monopoly case, and ISP can estimate aggregate demand and choose a site-to-site price equal to its site-to-site cost (see my PhD thesis)

- For the oligopoly case:
  - Characterise equilibria for a given solution concept
    - How quickly can the system converge to these?
    - How stable are these?

Thank You!

Questions?