Two-layer network design by discrete optimization

Arie Koster

Warwick Business School

joint work with Sebastian Orlowski, Christian Raack (both Zuse Institute Berlin), and Nokia Siemens Networks

Centre for Discrete Mathematics and its Applications – DiMAP

WARWICK BUSINESS SCHOOL

THE UNIVERSITY OF WARWICK

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The Project

- **Goal:** Speed-up the design of (almost) optimal multi-layer network designs
- **Methodology:** Problem-specific enhancements of integer linear programming solvers

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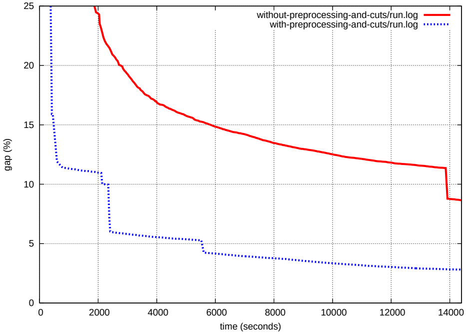


Gap vs. time
with/without cutting planes

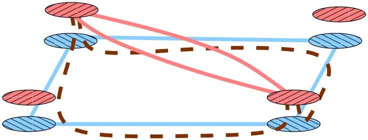
- **Achievements:** Significant performance improvement by cutting planes
  - Increase of lower bound on total cost
  - Faster achievement of desired quality guarantee

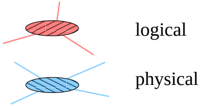
Outline of the talk

- Two-Layer Network Design
- Mathematical Model
- Solving ILPs
- Cutting planes
- Computational results
- Summary and Outlook

Two-Layer Network Design

Two-layer planning problem





Objective: Minimum cost network design

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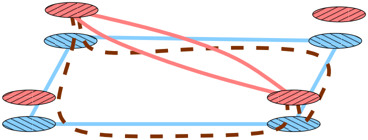
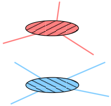
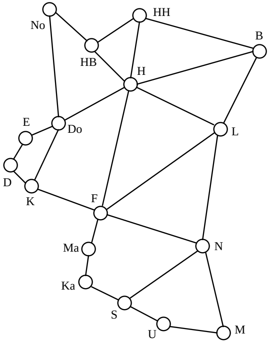




Objective: Minimum cost network design

- Single fiber physical WDM network with 80 channels/fiber



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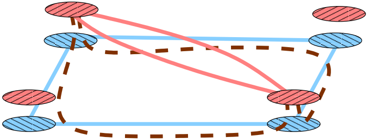

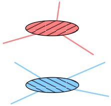

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- Single fiber physical WDM network with 80 channels/fiber
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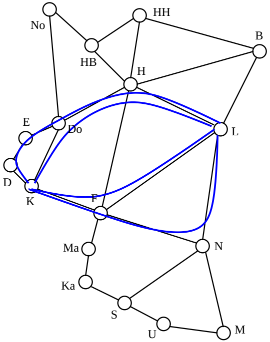
- Lightpaths installable at different bitrates (2.5, 10, or 40 Gbit/s)



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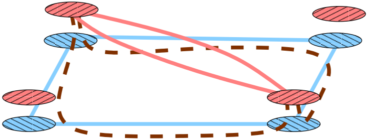

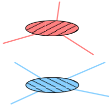

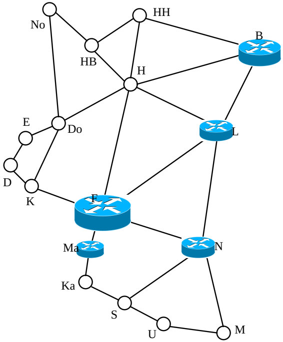

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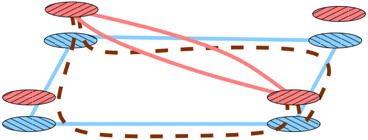

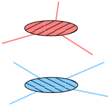

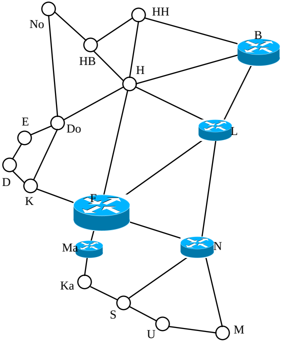

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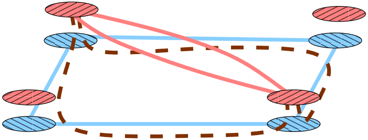

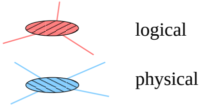

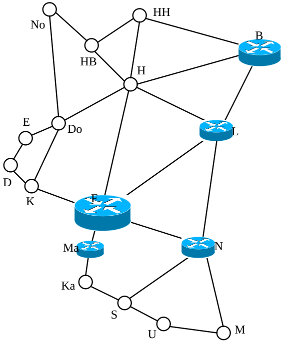

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- EXCs installable at nodes
- Lower granularity point-to-point demands
- (Survivability against node/link failures by 1+1 protection)

Mathematical model

Minimize Node cost + Logical link cost + Physical link cost

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subject to

- Routing of demands at logical link layer
- Installation of enough lightpaths on logical links
- Installation of enough EXC capacity at nodes
- Installation of physical links, number of lightpaths per fiber
- (Diversification of routing for protected demands)

Mathematical model – Objective & Demands

Objective: Minimization of total network cost

$$\min \sum_{i \in V} \sum_{m \in M_i} c_i^m x_i^m + \sum_{\ell \in L} \sum_{m \in M_\ell} c_\ell^m y_\ell^m + \sum_{e \in E} c_e x_e \qquad (1)$$

where

- nodes V, EXCs $m \in M_i$ at cost c_i^m, variable $x_i^m \in \{0, 1\}$
- logical links L, modules $m \in M_\ell$ at cost c_ℓ^m, variable $y_\ell^m \in \mathbb{Z}_+$
- physical links E, setup cost c_e, variable $x_e \in \{0, 1\}$

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Demands & Commodities

- Demand: point-to-point request
- Commodity: aggregation of demands with same source

Mathematical model – Flow & Logical link capacities

Flow conservation:

$$\sum_{j \in V} \sum_{\ell \in L_{ij}} (f^k_{\ell,ij} - f^k_{\ell,ji}) = v^k_i \qquad i \in V,\ k \in K \tag{2}$$

- K set of commodities (aggregated point-to-point demands)
- v^k_i demand value of $k \in K$ at node $i \in V$
- L_{ij} logical links connecting i and j, $f^k_{\ell,ij} \geq 0$ flow from i to j along ℓ

Logical link capacities:

$$\sum_{k \in K} f^k_\ell \leq \sum_{m \in M_\ell} C^m_\ell y^m_\ell \qquad \ell \in L \tag{3}$$

where $f^k_\ell = f^k_{\ell,ij} + f^k_{\ell,ji}$ and C^m_ℓ capacity (bitrate) of lightpath-type $m \in M_\ell$

Mathematical model – Node capacities

EXC selection:

$$\sum_{m \in M_i} x_i^m \leq 1 \qquad i \in V \tag{4}$$

At most one EXC can be installed at every node $i \in V$

EXC switching capacity:

$$\underbrace{\sum_{\ell \in L_i} \sum_{m \in M_\ell} C_\ell^m y_\ell^m}_{\text{Logical link capacity}} + v_i \leq 2 \underbrace{\sum_{m \in M_i} C_i^m x_i^m}_{\text{EXC Capacity}} \qquad i \in V \tag{5}$$

where $v_i := \sum_{k \in K} |v_i^k|$ emanating demand at node $i \in V$

Mathematical model – Physical link capacities

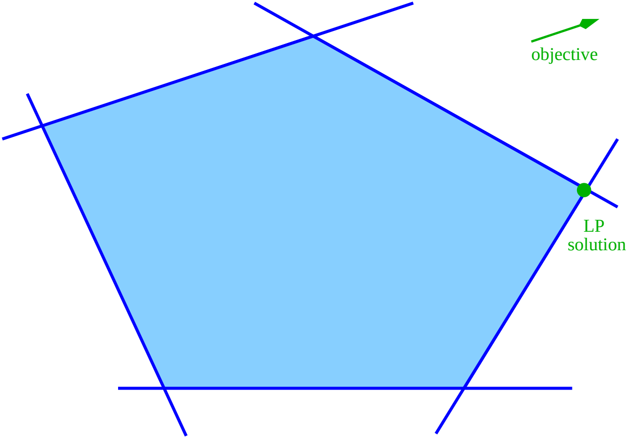
Physical link capacity:

$$\sum_{\ell \in L_e} \sum_{m \in M_\ell} y_\ell^m \leq B_e x_e \qquad e \in E \tag{6}$$

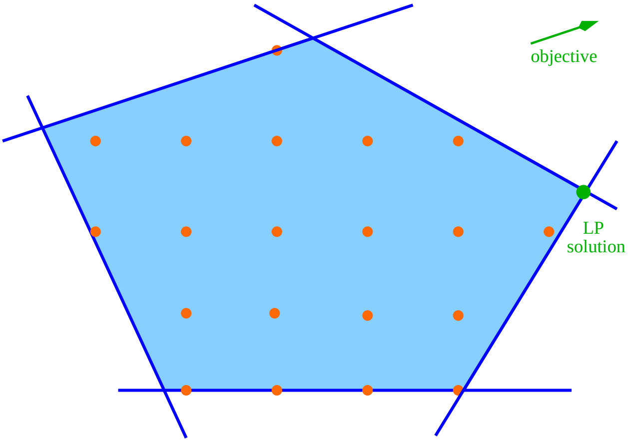
where

- B_e capacity of $e \in E$ (# wavelengths)
- L_e set of logical links containing physical link $e \in E$

Solving Integer Linear Programs



- Linear programs can be solved efficiently (in theory and practise)



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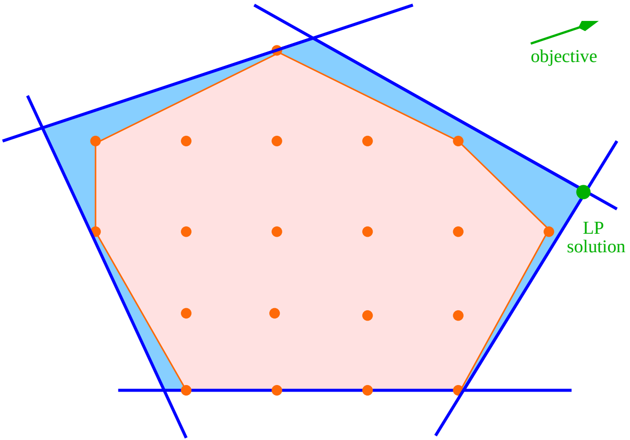
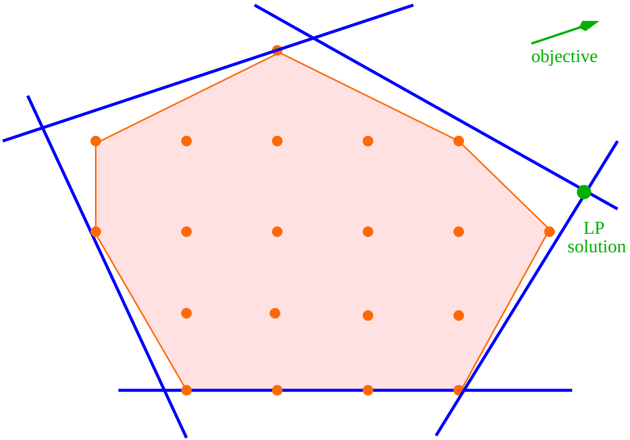
Integer Linear Programming



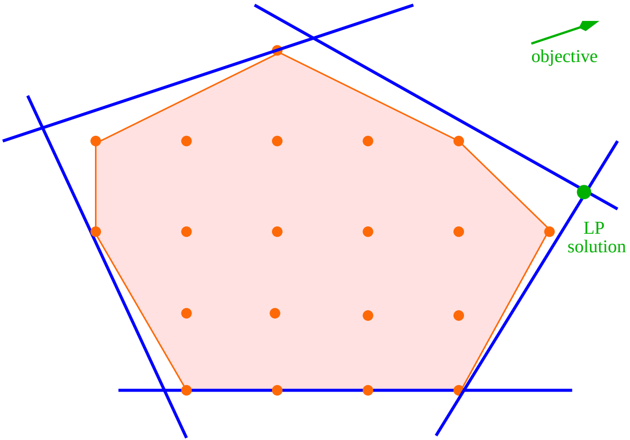
- Linear programs can be solved efficiently (in theory and practise)
- (Mixed) Integer linear programs are harder to tackle
- Knowledge about convex hull of integer solutions is needed

Cutting planes



- Solution to LP relaxation is not part of the convex hull

Cutting planes



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- Explore problem structure: valid inequalities

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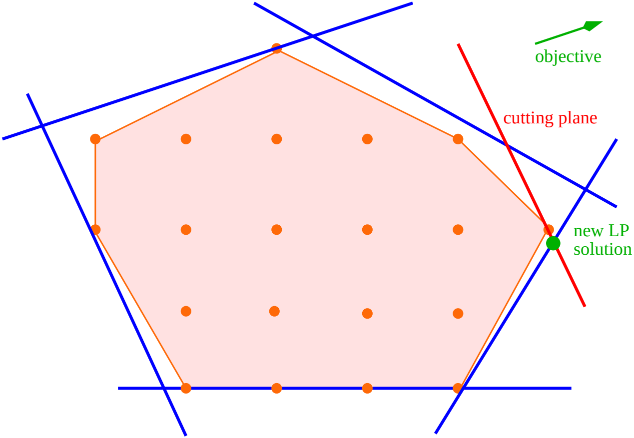
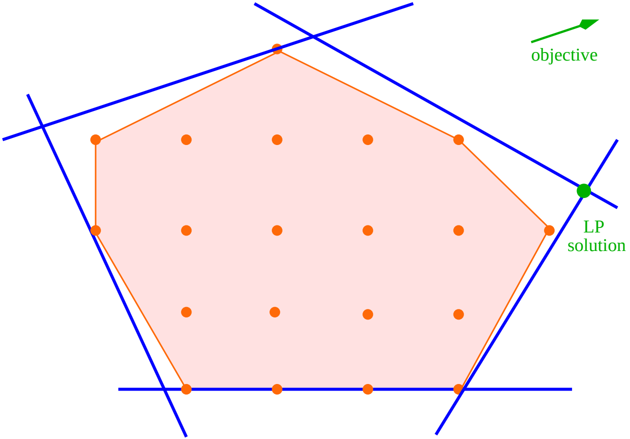
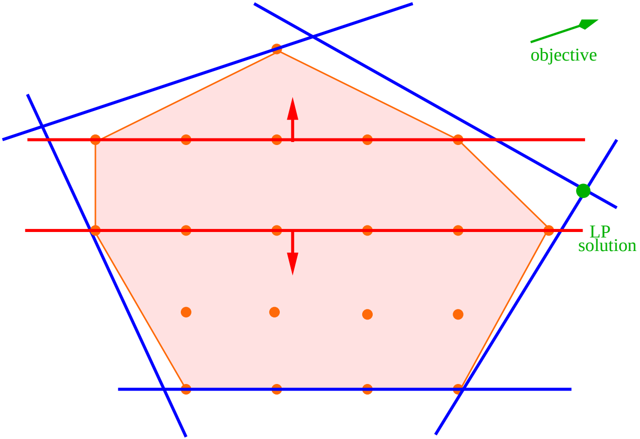


- Solution to LP relaxation is not part of the convex hull
- Explore problem structure: valid inequalities
- Add violated inequality to LP relaxation: cutting plane

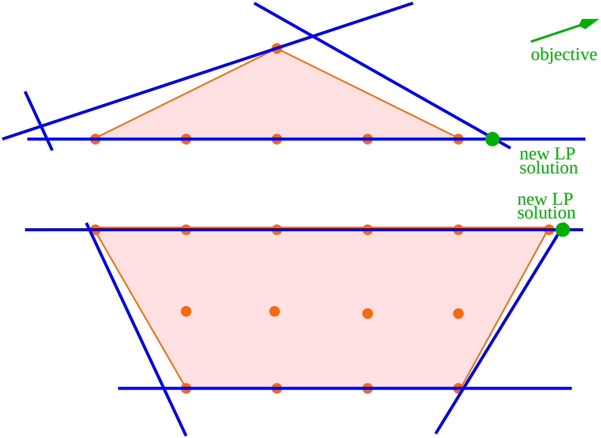


- Solution to LP relaxation is fractional in some variables



- Solution to LP relaxation is fractional in some variables
- Branch on fractional value of integer variable

Branch & Bound / Cut



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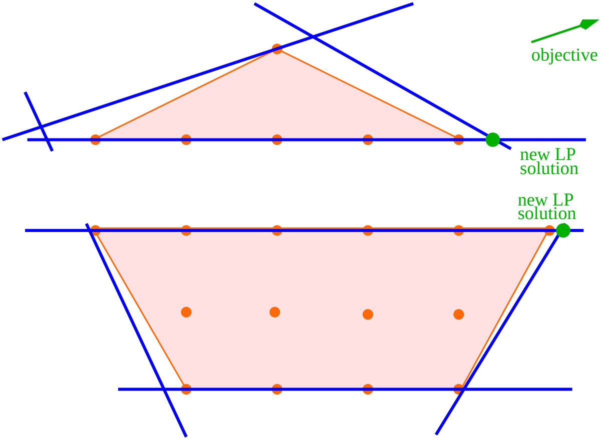
- Bound solution space by best solution

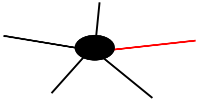


- Solution to LP relaxation is fractional in some variables
- Branch on fractional value of integer variable
- Bound solution space by best solution
- Cutting plane + Branch & Bound: Branch & Cut

Problem-specific cutting planes

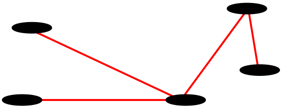
Physical fixed-charge cuts

Physical degree constraint for demand end-nodes $i \in V$

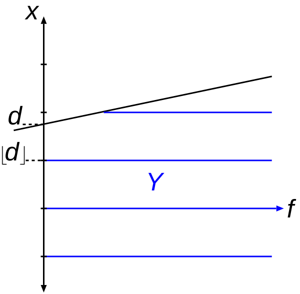


$$\sum_{e \in \delta(i)} x_e \geq 1$$

Physical tree between all demand end-nodes



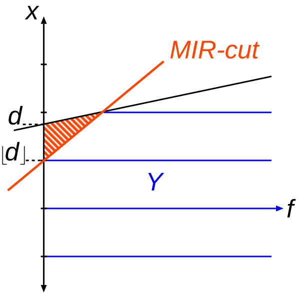
$$\sum_{e \in E} x_e \geq |V| - 1$$



Base inequality: $af + x \leq d$

where $f \in \mathbb{R}_+, \; x \in \mathbb{Z}_+$

Mixed-Integer Rounding (MIR)



Base inequality: $af + x \leq d$

where $f \in \mathbb{R}_+,\ x \in \mathbb{Z}_+$

MIR inequality: $\frac{a}{1-<d>} f + x \leq \lfloor d \rfloor$,

where $< d >= d - \lfloor d \rfloor$

Note: MIR introduces integral vertices!

$(f, x) \in \mathbb{R}^m \times \mathbb{Z}^n$

Base: $$\sum_{i=1}^{m} a_j f_j + \sum_{j=1}^{n} c_j x_j \geq d$$

MIR: $$\sum_{i=1}^{m} \bar{F}_{d,c}(a_j) f_j + \sum_{j=1}^{n} F_{d,c}(c_j) x_j \geq F_{d,c}(d)$$

where

$$F_{d,c}(a) := r(d,c)\lceil a \rceil - (r(d,c) - r(a,c))^+$$
$$\bar{F}_{d,c}(a) := r(d,c) a^+ = \lim_{t \searrow 0} F_{d,c}(at)/t$$

Details: see Nemhauser/Wolsey 1988, Raack 2005

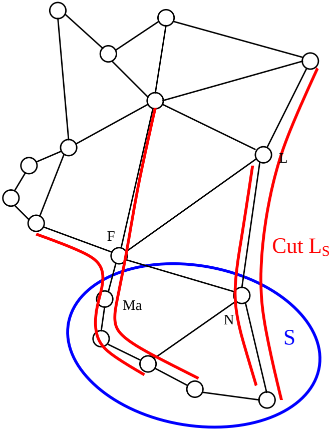
Important property of MIR functions

- Base inequality valid $\Rightarrow$ MIR inequality valid

Problem-specific MIR-based inequalities

- Construct base inequality $\underbrace{a^T f}_{\text{flow}} + \underbrace{c^T x}_{\text{capacity}} \geq \underbrace{d}_{\text{demand}}$
- For all capacity coefficients c_j:
  - Compute F_{d,c_j}-MIR inequality and test it for violation.
- Application: MIR cutset, Flow-cut inequalities

MIR cutset inequalities



Base inequality

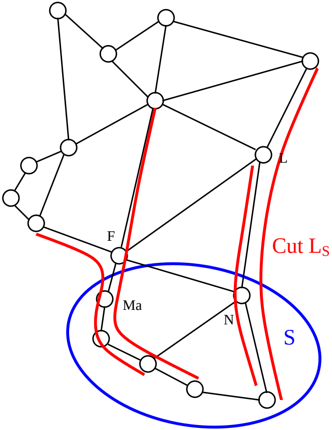
Given: Cutset L_S on the logical layer

Base inequality: cut capacity $\geq$ demand

$$\sum_{\ell \in L_S} \sum_{m \in M_\ell} C_\ell^m y_\ell^m \geq d_S$$

MIR inequality for each $c = C_\ell^{m_0}$:

$$\sum_{\ell \in L_S} \sum_{m \in M_\ell} F_{d_S,c}(C_\ell^m) y_\ell^m \geq F_{d_S,c}(d_S)$$



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Finding violated inequalities

- $\mathcal{NP}$-hard, graph shrinking heuristic

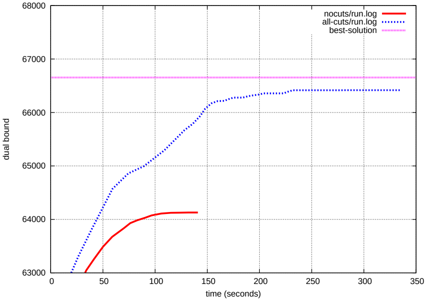
Computational results

Cutting planes: computational results

Computations with SCIP (http://scip.zib.de)

17-node German network, no physical fixed-charge cost:

Lower bound over time (root):



Close-to-optimal lower bound

by cutting planes

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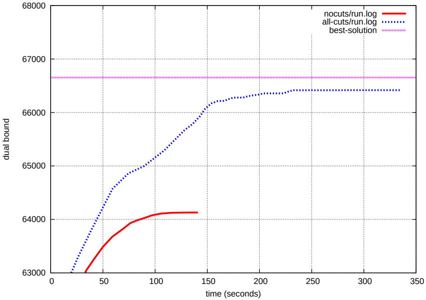
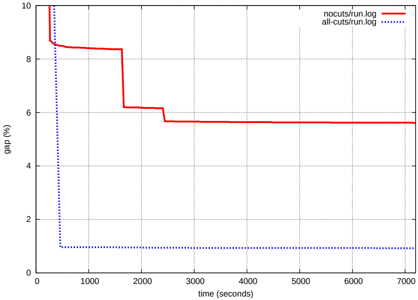


Close-to-optimal lower bound by cutting planes

Optimality gap over time (2h):



1% optimality gap within 5 min instead of 5.6% after 2h!

Cutting planes: computational results

17-node German network with physical fixed-charge cost:

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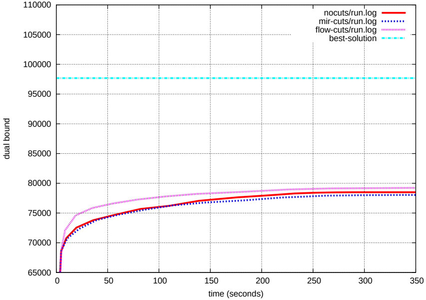

Logical layer MIR- and Flow-cuts:

$\Rightarrow$ only slight improvement

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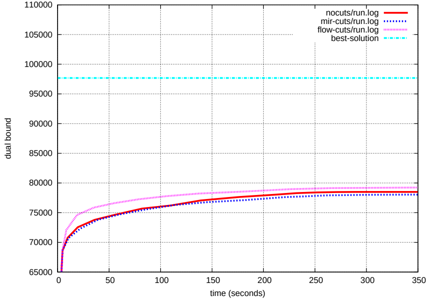

Logical layer MIR- and Flow-cuts:

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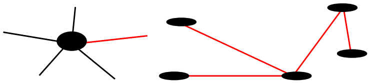
LP observations

- Highly fractional x_e variables
- Gap from physical layer!

⇒ Use degree and tree cuts!



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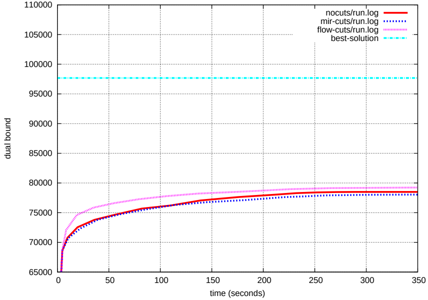

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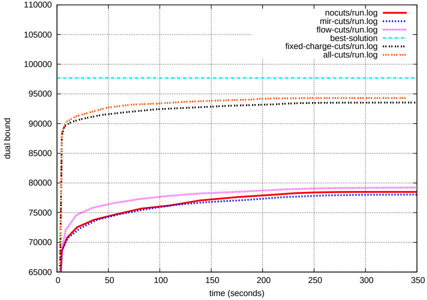

Logical layer MIR- and Flow-cuts:

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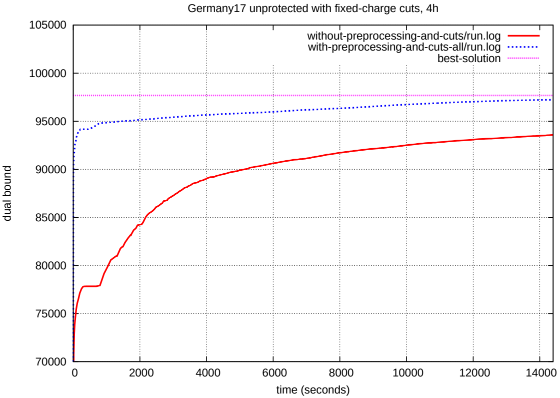
Also physical-layer cuts needed!

Lower bound over time with all cuts (4 hours):



Close-to-optimal lower bound!

Summary and Outlook

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Further reading: ZIB-Report 07–XX – coming soon

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joint work with Sebastian Orlowski, Christian Raack (both Zuse Institute Berlin), and Nokia Siemens Networks

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