On simple routing games

Modelling Interdomain routing with game theory

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Outline

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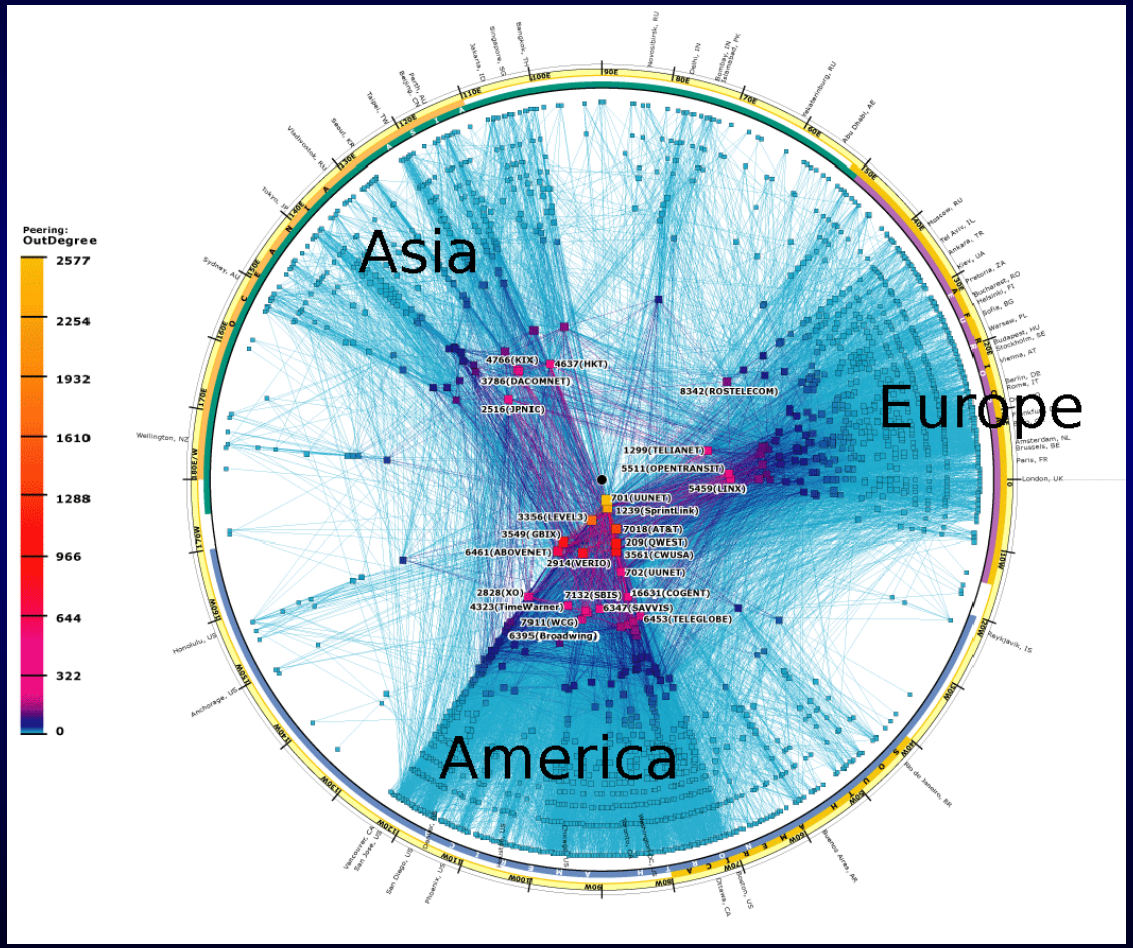
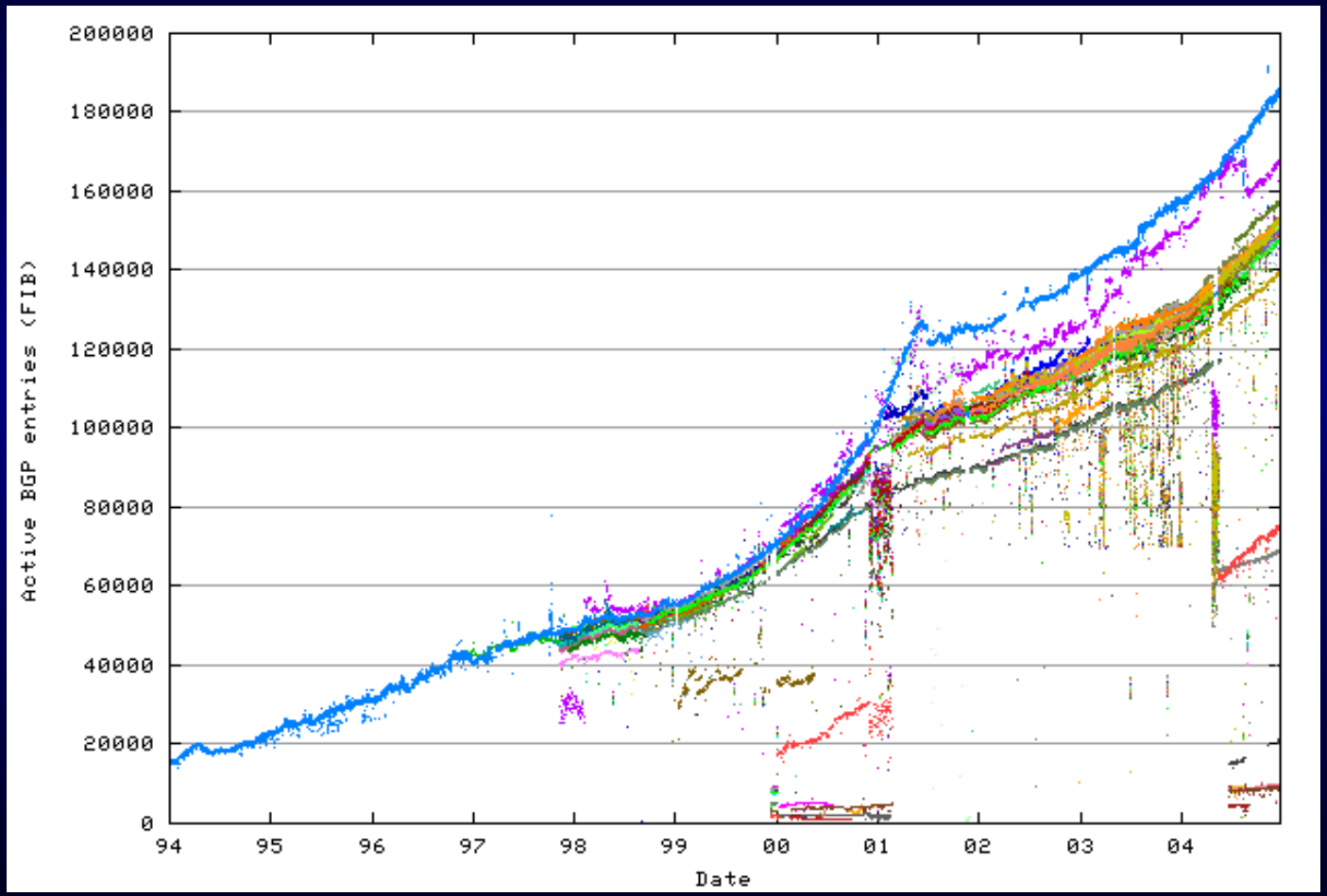
- ASes communicate over the *Border Gateway Protocol* (BGP).

How does the AS graph look like?



BGP growth



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Informally, the stable path problem is a routing problem on an undirected graph with all nodes trying to reach a single destination ‘0’. Each node has a ranked list of preferred paths to ‘0’. A solution to the stable path problem is a set of paths, one for each node, where each node is assigned the highest ranked path which does not conflict with paths chosen by other nodes.

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- If the above is not true, the game is called *Bayesian*.

Nash Equilibria

Definition

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- A Nash equilibria always exist in mixed strategies.
- Sometimes, the Nash equilibrium results in the worst possible payoffs as in the *Prisoners' dilemma*.

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3. The payoff function depends on rank of the path (strategy) chosen and the ability to successfully route to AS 0.
4. The game is of full information such that all players know the rules of the game, their payoffs and the set of strategies of all other players. If the game is played multiple times, all the players know all the past moves of all the players.

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Note that this argument is really the inverse of the proper one: BGP has been shown to be unstable, hence all model of BGP must show the potential to be unstable. . .

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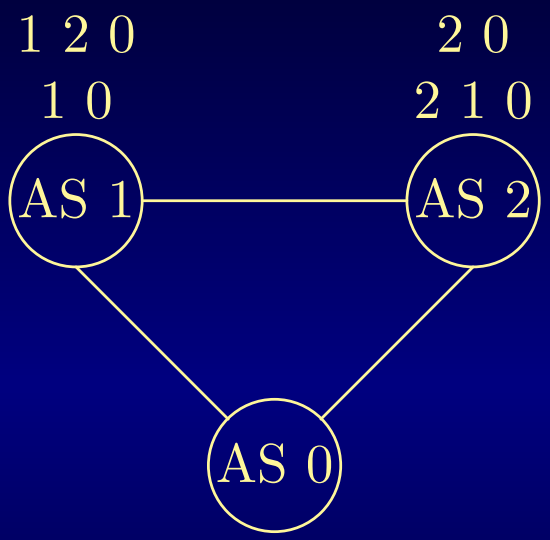

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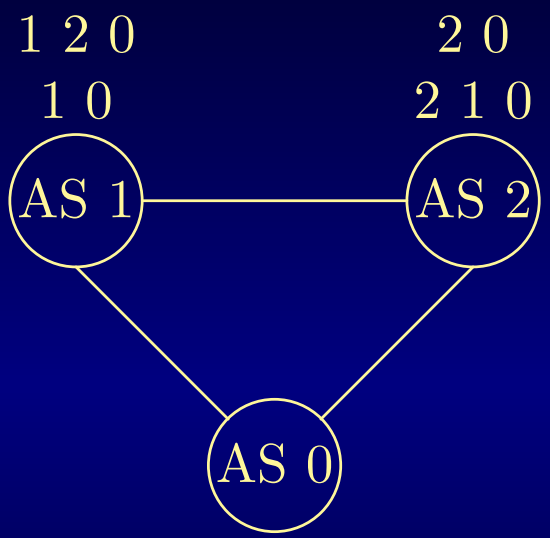

	AS 2	
AS 1	first	second
first	$2, 2$	$-2, -2$
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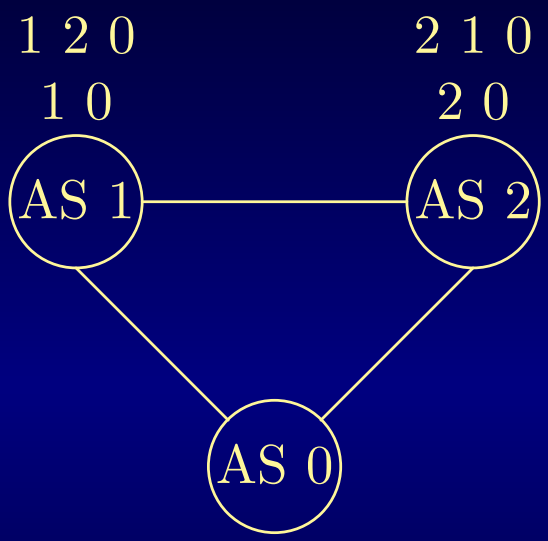

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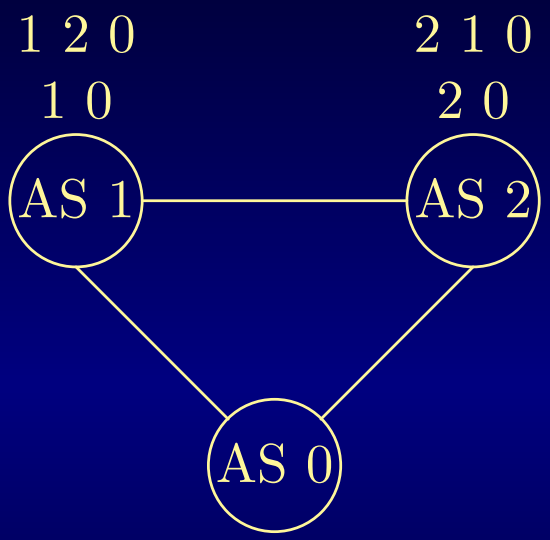

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Baysian routing games

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A Bayesian routing game follows

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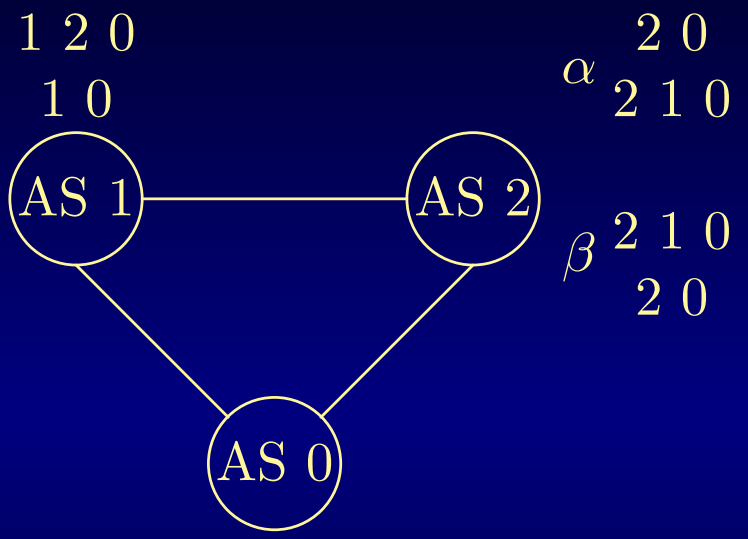

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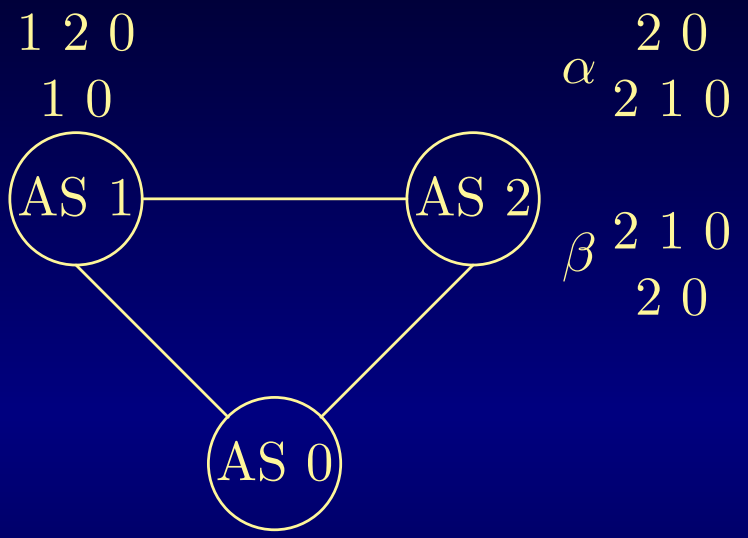

		AS 2		
	α		β	
AS 1	first	second	first	second
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A classical example is the n-armed bandit, a slot machine with n levers.

Fictitious play

A *fictitious play* strategy is a process by which players assume that the strategies of their opponents are randomly chosen from some unknown stationary distribution. In each period, a player selects his best response to the historical frequency of actions of his opponents.

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Would cooperative game theory help? The short answer is no: ASes are generally competing companies. The real answer is maybe: Some ASes do enter into trade agreements. So, it is possible for some AS to cooperate with some of its neighbours and not with others.

. . . and finally

This is the end!

This work was funded by Nortel Networks and is currently used as a foundation work for an EPSRC proposal.

Thank you very much for your attention.

These slides are available on *http://gridlock.york.ac.uk/staff/yann* in both PDF and metapost with LaTeX. This presentation was done using *Latex Beamer* and LaTeX.